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ORBITAL DISPERSION SIMULATION OF NEAR-EARTH OBJECT DEFLECTION/FRAGMENTATION BY NUCLEAR EXPLOSIONS

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ABSTRACT

Attempts to deflect a near-Earth object (NEO) from an impact trajectory using high-energy methods, such as nuclear explosions, can plausibly fragment the NEO rather than deflect it. This paper addresses the orbital dispersion problem of a fragmented NEO using the asteroid 99942 Apophis as a model, and presents a physical description of the relative motion phenomena of a fragmented NEO. The overall stochastic model is approximated using a quasi-random process in which the randomness of fragmentation is associated only with the initial condition. Categories of initial probability distributions are analyzed to obtain relationships between initial conditions and eventual percentage of impacting bodies. A Direct Simulation Monte Carlo approach is used to further investigate the sample initial conditions for the Apophis example previously studied in the literature. Consequences of ignoring mutual gravitation of the fragmented system or relying on the linearized Clohessy-Wiltshire-Hill model for large time scales are discussed.

1. Introduction

There is evidence that the impact of a near-Earth object (NEO) would constitute a significant threat to life on Earth. With a large number of such bodies crossing the orbit of Earth, the probability of a catastrophic impact is large enough to warrant careful monitoring of hazardous NEOs and research into the possible deflection of their orbit [1-2]

Deflection methods of sufficiently high energy density are required, and need to be prepared in advance of an expected impact date with the Earth. One of these methods is the use of nuclear explosive devices above, on, or beneath the surface of an NEO [3-5]. While these methods are intended to ablate a thin layer of material, the level of energy imparted to the NEO makes fragmentation of the target a plausible outcome [5-6]. The specific energy added to the basaltic rock can approach or even exceed the energy needed for minimal break up of a small NEO [6]. In other cases, thermal ablation of surface material causes compression waves to propagate through the remainder of the NEO. The stress of these waves may be enough to continue fracturing the material [7].

It has been suggested that fracturing a body may be a beneficial outcome, as smaller pieces may burn up in the atmosphere. However, even smaller pieces impacting the Earth can cause significant damage [8]. The lower threshold for material ablation in the atmosphere is undetermined, and explosions in the atmosphere can be fatal and possibly catastrophic events [8]. A possible benefit of NEO fragmentation is to lower the number of small bodies impacting the Earth in cases where some level of impact is inevitable. This could be either the result of an unsuccessful deflection attempt or a backup measure when there is not enough time for another deflection mission. Previous research has shown a reduction in impacting mass of up to 80% through statistical methods [6]. Lead time, or warning time, has been suggested as the most important factor in the effectiveness of catastrophically fragmenting a NEO [6-7].

A Direct Simulation Monte Carlo approach is used to further investigate an illustrative example of asteroid Apophis previously studied in [6]. This is contrasted to the results of their statistical state-space method by comparing the mean of the result for several simulations using the same initial condition parameters. Consequences of ignoring mutual gravitation of the fragmented system or relying on the circular Clohessy-Wiltshire-Hill model for large time scales are discussed in this paper. We conclude that the present model including gravitational interactions leads to significantly different results, and that previous research needs to be adapted to address a shifting reference orbit for large time scales.

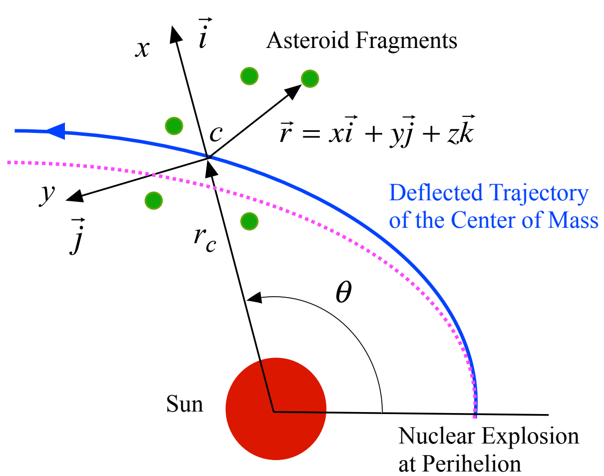

Figure 1: Illustration of the rotating LVLH reference frame in an elliptical orbit. The *z*-axis is perpendicular to the nominal orbital plane

2. Problem Formulation

2.1 Clohessy-Wiltshire-Hill Equations

Since keeping track of particles from a fragmented NEO becomes difficult on the scale of heliocentric orbits, we can gain higher resolution by keeping track of the particles relative to the unperturbed orbit of the NEO. Therefore, integrating the equations of motion for the system is necessary only for the deviation from the reference orbit. Figure 1 shows the rotating reference frame associated with the NEO orbit, often called the local-vertical-local-horizontal (LVLH) reference frame [9].

The position vector of a body with respect to the origin of the LVLH reference frame is expressed as

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \tag{1}$$

where (x, y, z) are the radial, transverse, and vertical components the position vector in the LVLH reference frame and $(\vec{i}, \vec{j}, \vec{k})$ are the basis vectors of the rotating LVLH frame. The relative orbital motion of a body can then be described by the so-called Clohessy-Wiltshire-Hill (CWH) equations for an elliptical reference orbit [9-10], as follows:

$$\begin{aligned} \ddot{x} - \left(\dot{\theta}^2 + 2\frac{\mu}{r_c^3}\right)x - \ddot{\theta}y - 2\dot{\theta}\dot{y} = 0 \\ \ddot{y} - \left(\dot{\theta}^2 - \frac{\mu}{r_c^3}\right)y + \ddot{\theta}x + 2\dot{\theta}\dot{x} = 0 \\ \ddot{z} + \frac{\mu}{r_c^3}z = 0 \end{aligned} \tag{2}$$

where μ is the gravitational parameter of the sun, r_c is the orbital distance of the origin of the LVLH frame from the sun, θ is the true anomaly, as indicated in Fig. 1. We also have the following nominal elliptical orbit equations:

$$\begin{aligned} \ddot{r}_c - \dot{\theta}^2 r_c + \frac{\mu}{r_c^2} = 0 \\ \ddot{\theta} + \frac{2\dot{r}_c\dot{\theta}}{r_c} = 0 \end{aligned} \tag{3}$$

For a circular reference orbit (or as an approximation of elliptical orbits on short time scales), we simply have the standard CWH equations of the form

$$\begin{aligned} \ddot{x} - 3n^2 x - 2n\dot{y} = 0 \\ \ddot{y} + 2n\dot{x} = 0 \\ \ddot{z} + n^2 z = 0 \end{aligned} \quad (4)$$

where n is the circular orbit rate, assumed to be constant. These equations have been successfully used to describe orbital debris problems from exploded satellites, and provide insight into the physical mechanism of orbital dispersion that makes a fragmented NEO spread out along its orbit with time [11]. Figure 2 shows the pattern of a debris cloud as a function of true anomaly for an isotropic distribution of initial relative velocity. After the first orbit, the debris cloud will look like simply stretching along the orbital path. It should be mentioned that the characteristics of the orbit are the governing factors behind this phenomenon, and that all initial distributions with material at a nonzero distance from the center of mass will result in similar cloud shapes.

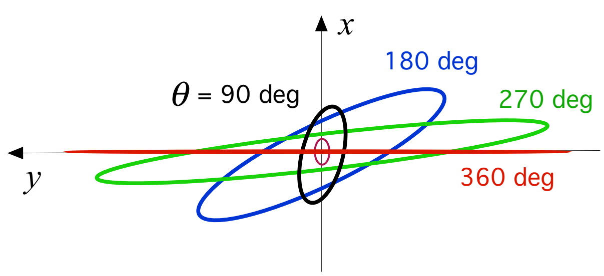

Figure 2: Debris cloud distortion with increasing true anomaly [6, 11].

2.2 Initial Conditions

In its general form, the problem of modeling a fragmented NEO is stochastic in nature. The initial conditions of the fragmentation are unknown, and the collisions cannot be accurately modeled as they can lead to further fragmentation, merging of bodies, or changes in rotation rates of the fragments. While a statistical description of the system can be formulated and solved in terms of percentage of impacts as discussed in [6], it does not allow for associations between the unknown initial conditions and the eventual percentage of impacting fragments. These associations would be desirable for any future impact minimization research.

For the preliminary study, a collision-less dynamical model of the fragmented NEO is assumed. The system is approximated by a quasi-random process. The randomness associated with the initial conditions can be used as a starting point [6], and the resulting system can be simulated as a deterministic model. Since the initial conditions are best described by probability density functions (PDFs) [6], a method of characterizing the behavior of the PDFs is desired. The algorithm used in this paper takes the PDFs of the position and velocity initial conditions as input. It then generates a set of position and velocity components that fit the PDFs and propagates them to determine the output of the model. The system dynamics can then be simulated, and the process is repeated and averaged to piece together the behavior of the initial PDF. This approach is called a Direct Simulation Monte Carlo (DSMC) method [12].

The PDF of the initial condition is assumed as

$$P(\mathbf{x}, \mathbf{v}; t) = \int_{-\infty}^{t} \rho(\mathbf{x}, \mathbf{v}; \tau)\, d\tau \quad (5)$$

where $\rho(\mathbf{x}, \mathbf{v}; t)$ is the probability of a fragment to have position $\mathbf{x} = (x, y, z)$ and velocity $\mathbf{v} = (\dot{x}, \dot{y}, \dot{z})$ at a time t [6]. The algorithm uses a discrete random variable ξ, chosen by the computer between 0 and 1, to optimize the following equation:

$$\zeta - \int_{\Gamma} P(\mathbf{x}, \mathbf{v}; t) \geq 0 \quad (6)$$

where Γ is the set in phase space with components less than **x** and **v**.

$$\Gamma = \left\{ \mathbf{r}, \dot{\mathbf{r}} \in \mathbb{R}^3 \,\middle|\, \mathbf{r}_i \leq \mathbf{x}_i, \dot{\mathbf{r}}_i \leq \mathbf{v}_i \right\} \quad (7)$$

This approach allows **x** and **v** to be chosen to fit the desired PDF. The mass distribution of the fragments is also something that has a significant effect on the system state evolution [12]. Several distributions of fragmented asteroids are expected, and the distribution of the mass is often governed by the largest piece remaining. The porosity and the composition of the asteroid can play a critical role in how the break up occurs. As a first approach to the problem, equally sized fragments are assumed to be distributed from the asteroid isotropically.

3. Results

3.1 Simulation Model

The equations of motion of fragmented bodies in the elliptical reference orbit, including the mutual gravitation among fragments are given by

$$\begin{aligned}
\ddot{x}_j &= \left(\dot{\theta}^2 + 2\frac{\mu}{r_c^3}\right)x_j + \ddot{\theta}y_j + 2\dot{\theta}\dot{y}_j \\
&\quad + \sum_{i \neq j} \frac{Gm_i m_j}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3}\left(x_i - x_j\right) \\
\ddot{y}_j &= \left(\dot{\theta}^2 - \frac{\mu}{r_c^3}\right)y_j - \ddot{\theta}x_j - 2\dot{\theta}\dot{x}_j \\
&\quad + \sum_{i \neq j} \frac{Gm_i m_j}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3}\left(y_i - y_j\right) \\
\ddot{z}_j &= -\frac{\mu}{r_c^3}z_j + \sum_{i \neq j} \frac{Gm_i m_j}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3}\left(z_i - z_j\right)
\end{aligned} \tag{8}$$

These relative equations of motion are propagated for each particle in the fragmented system, including the gravitational interaction among the particles, using a 4^{th} order Runge-Kutta method. A lower bound is added to the positional distance for the gravity calculations in order to keep the collision-less nature of the problem from leading to non-physical accelerations. This addition deviates from a true n-body gravitational simulation, but ensures that the results conserve energy for the system and reduces round-off error.

3.2 Illustrative Cases

The case of Apophis fragmenting into small, uniformly-sized bodies is considered here. This would illustrate what would happen if a NEO with similar orbital characteristics to Apophis, but significantly higher porosity, would fragment due to a deflection attempt exceeding the gravitational binding energy [4, 13].

Asteroid 99942 Apophis, previously known by its provisional designation 2004 MN4, was discovered on June 19, 2004. Apophis is an Aten-class asteroid with an orbital semi-major axis less than 1 AU. Its mass is estimated to be 4.6e10 kg and its size is estimated to be about 270 m in diameter. It has an orbital period of 323 days about the sun. After its close flyby of the Earth in 2029, it will become an Apollo-class asteroid. It is currently predicted to swing by at around 32,000 km from the Earth's surface in 2029 with a probability of 1 in 45,000 for a keyhole passage in 2029 to result in a resonant return to impact the Earth in 2036. Keyholes are very small regions of the first encounter b-plane such that if an asteroid passes through one, it will have a resonant return impact with the Earth. The orbital parameters of Apophis are listed in Table 1.

Orbital Parameters	Value
Semi-major axis, *a* (AU)	0.92243
Eccentricity, *e*	0.19120
Inclination, *i* (deg)	3.33142
Argument of perihelion, *ω* (deg)	126.404
Longitude of right ascension, *Ω* (deg)	204.442
Mean anomaly at epoch, *M*o (deg)	117.468

Table 1: Orbital elements of asteroid Apophis at epoch 2455000.5 (2009-June-18.0) TDB. Source: JPL's small-body database.

The case of an undeflected reference orbit is investigated, as well as the case where a velocity change of 2 cm/s is added to the tangential velocity of the center of mass in order to compare to the previous results of [6]. After the completion of the simulation, a flyby of the Earth, including Earth's gravitational model, is used to determine the percentage of impacting mass. The simulation was run 30 times for each set of initial condition parameters, and the results averaged to determine the expected outcome.

Figure 3 shows the distribution of 10^7 kg fragments after 1 year of dispersion, in the undeflected case. The total number of fragments was 4600. The same initial conditions are shown after 5 years in Fig. 4. The stretching of the debris cloud over time can be seen in the difference between the two figures.

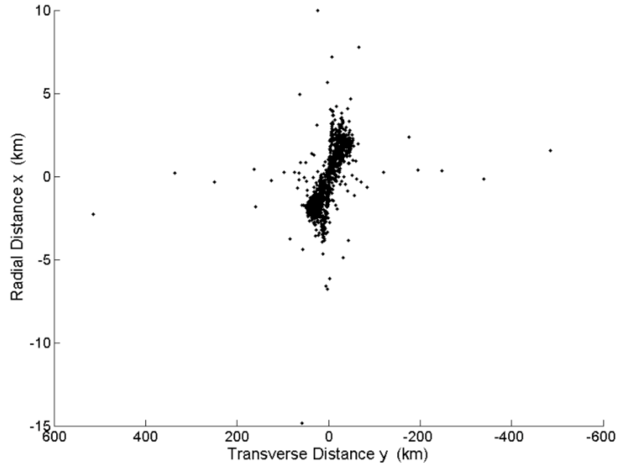

Figure 3: Sample distribution after 1-year orbital dispersion, for undeflected circular CWH equations.

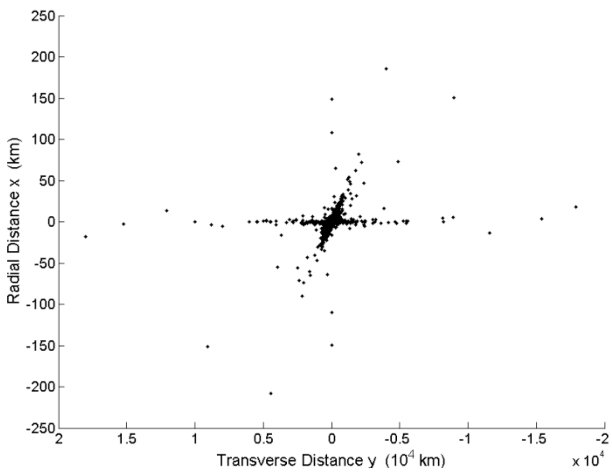

Figure 4: Sample distribution after 5-year orbital dispersion, for undeflected circular CWH equations.

The benefit of the DSMC approach as applied to this problem is that relationships between the initial condition parameters and the impacting mass (or impact percentage) can be established. Figures 5-8 show the relationships between total impacting mass after 5 years and initial position mean, initial position standard deviation, initial velocity mean, and initial velocity standard deviation. In each graph the other variables remain constant.

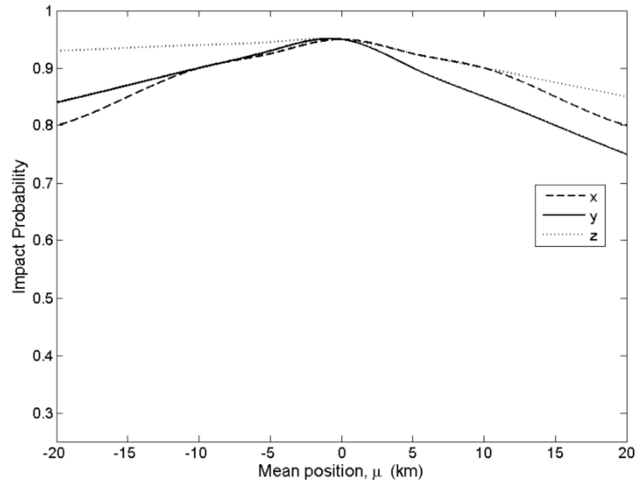

Figure 5: Impact probability dependence on mean position.

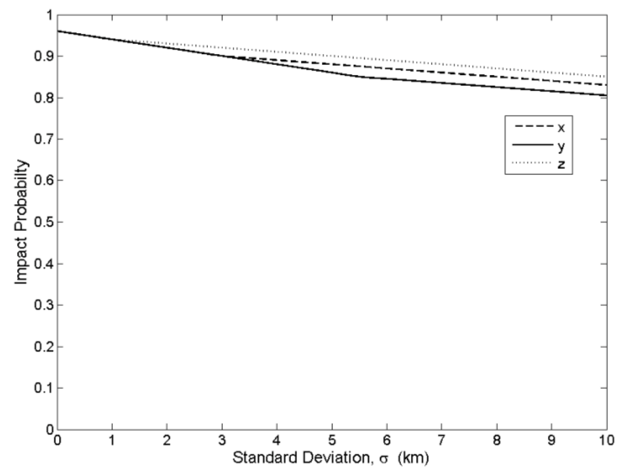

Figure 6: Impact probability dependence on position standard deviation.

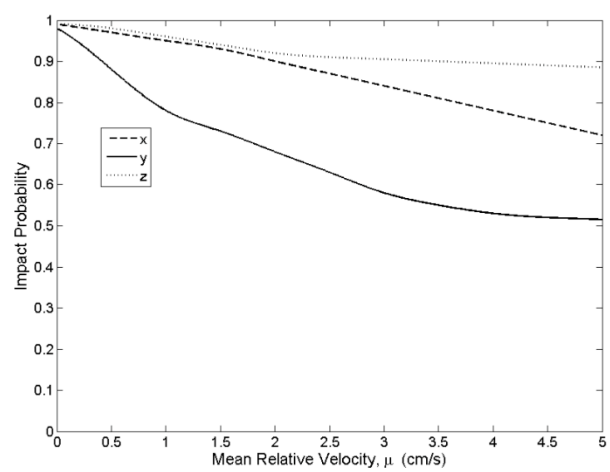

Figure 7: Impact probability dependence on mean relative velocity

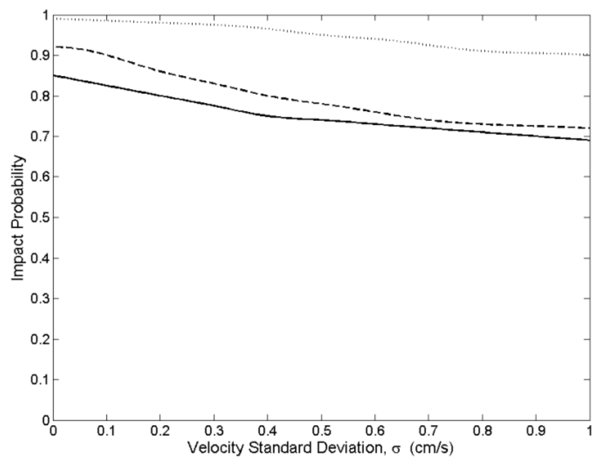

Figure 8: Impact probability dependence on relative velocity standard deviation.

In order to compare to previous results described in [6], the time dependence on orbital dispersion is shown for up to 5-year lead time. An elliptical reference orbit solution is shown to validate the model without the relative gravitational terms. It can be seen in Fig. 9 that there is good agreement between the proposed approach and the statistical model of the system used in [6]. The same calculation is completed including mutual gravitational terms, and is shown to lower the predicted impacting likelihood of fragmented bodies.

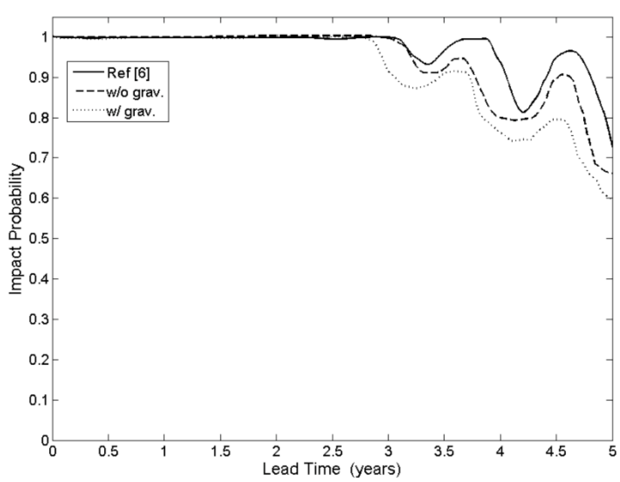

Figure 9: Comparison of current model to Ref. 6 with and without gravitational terms.

3.3 Circular CWH Comparisons

Due to the relative ease of computation for the circular CWH equations, it is desirable to understand the positional and velocity inaccuracies introduced by this model, and to identify the areas in which it may be used as a reasonable approximation. The average impact percentage as a function of time is shown in Figs. 10 and 11, contrasting circular and elliptical models both with and without gravitational calculations.

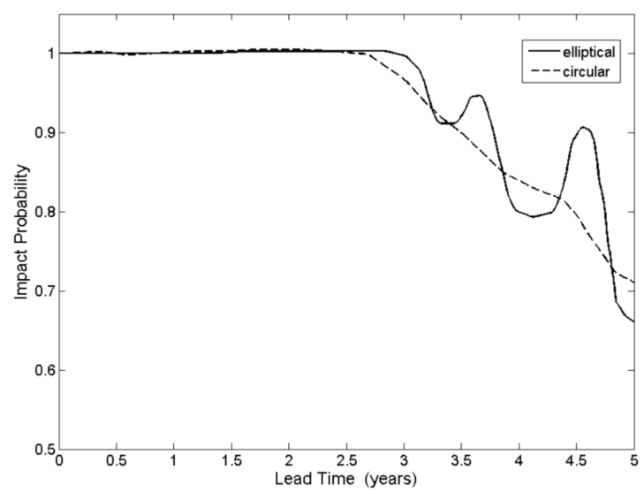

Figure 10: Comparison of circular and elliptic CWH models (without gravitational terms).

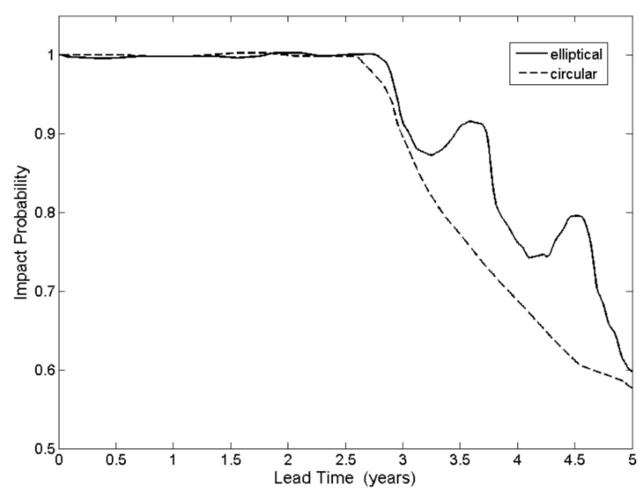

Figure 11: Comparison of circular and elliptic CWH models (with gravitational terms).

3.4 Importance of Mutual Gravitational Terms

In order to lower the number of calculations necessary to describe the relative orbits of NEO fragments, a typical assumption of previous work has been that gravitational interactions between the pieces of the fragmented body are significantly less than that of the sun, and can be neglected. One result of the present model is the ability to contrast the results of the deterministic process of orbital dispersion with and without gravitational interaction. It has been found that neglecting the gravitational interaction terms leads to positional errors large enough to influence the average total mass impacting the Earth. This effect is increased for larger particle sizes, and therefore these terms should only be neglected in cases where catastrophic disruption of the NEO occurs. In this case the problem is truly that of a debris cloud as described in [11].

Figure 12 shows the average absolute error associated with neglecting gravitational terms for fragment sizes of 10^5, 10^6, and 10^7 kg. The increase of this error with time should be noted, as most feasible scenarios for deflecting and fragmenting NEOs involve long periods of orbital dispersions.

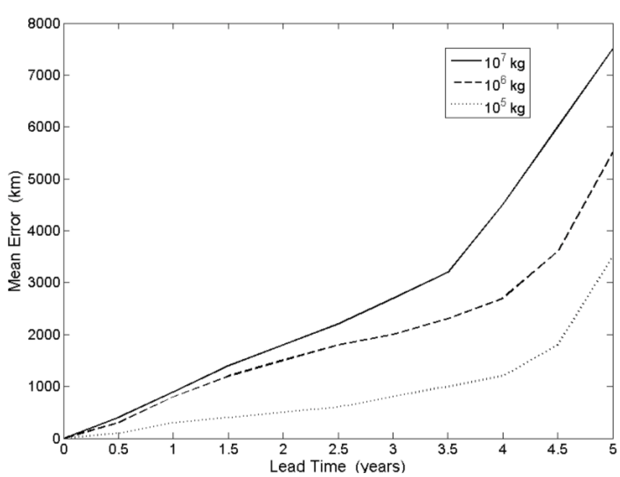

Figure 12: Mean position error from neglecting gravitational effects for 3 particle masses.

More important than the average absolute error introduced is the change in the dynamics of the system. The PDF of the dispersed system is altered by the inclusion of gravitational terms. Initial conditions with normal distributions are not preserved through the propagation process. While in the mutual-gravity-free case the shape of the distribution remains similar with only the parameters and variance changing, including mutual gravitation leads to two main dynamical regions where separate phenomena dominate. Unstable equilibrium points can be identified as the boundary at which gravitational contraction dominates. Outside of these points, the accelerations provided by the gravitational terms add to the rotational momentum of the system, spreading it out along the local orbital tangent. A qualitative description of these cases can be seen in Figs. 13 and 14.

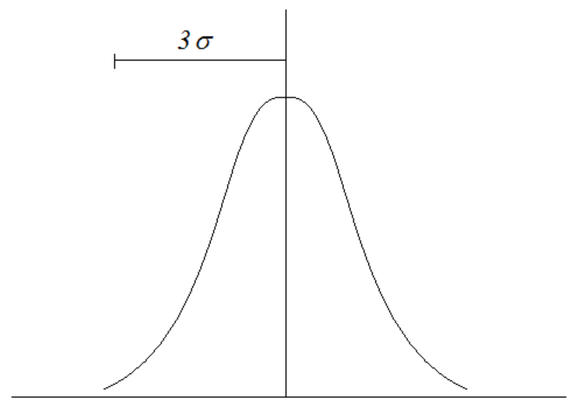

Figure 13: Example of probability density function in unperturbed case.

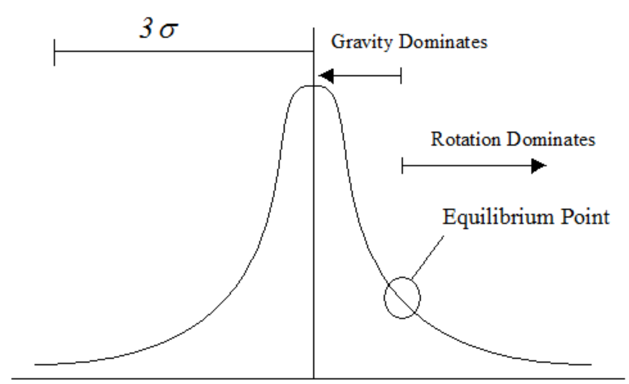

Figure 14: Example of probability density function with gravitational perturbations.

A method for quantifying this problem is to investigate the change in total distance from the center of mass caused by including gravitational accelerations. Figure 15 shows this calculation for 5 years of orbital dispersion with a Gaussian initial position distribution. A sign change in the comparison to the non-gravitational case can be seen in both transverse directions from the center of mass. Inside of this sign change, gravitational terms move particles closer to the center of mass. Outside of these sign changes, gravitational terms spread the particles away from the center of mass.

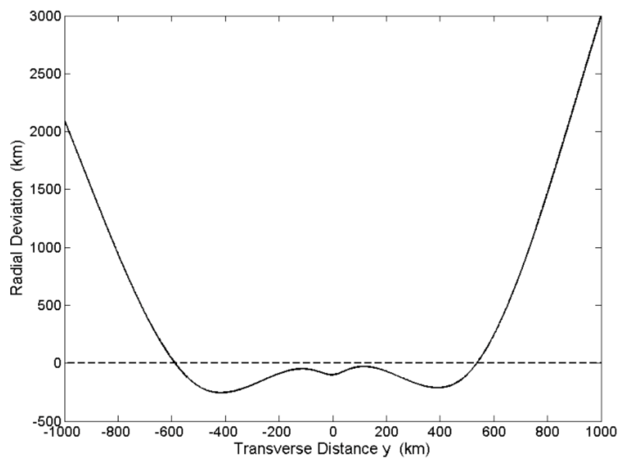

Figure 15: Deviation in radial distance from center of mass when neglecting gravitational terms.

4. Discussions

The general equations of relative motion of fragmented bodies in an elliptical reference orbit provide an adequate framework for the investigation of orbital dispersion of a fragmented NEO. Using the standard CWH equations for a circular reference orbit can provide a first-order approximation of how the system behaves on short time scales. This does not require keeping track of the time-varying orbital parameters, but provides increased acceleration errors for long periods of orbital dispersion.

The randomness of the orbital dispersion problem can be modeled by introducing the concepts of a probability density function (PDF) for the initial conditions. The resulting deterministic process can then be directly simulated to piece together the average behavior of many systems meeting the initial PDFs. As a limit process, the behavior should approximate the initial distribution, with a mean that does not deviate significantly by including additional cases.

Including the mutual gravitational interaction of the fragments can help to address errors in previous approaches, and better address the dynamics of the system. This introduction of computational complexity requires highly parallel algorithms to efficiently process model cases, and opens the door for using high-performance computing architectures for the design and optimization of missions for the successful deflection and fragmentation of NEOs.

Future work extrapolating the interactions between the various initial condition parameters is needed in order to effectively identify classes of initial conditions that minimize the total mass of particles impacting the Earth. Since the model presented here is capable of taking arbitrary PDFs as input for initial position, initial relative velocity, and initial mass distribution of fragments, further investigation of the results for non-normal initial distributions is desired. Specifically, the cases of a single large fragment dominating the dynamical model, and a small number of equally large fragments are currently being further investigated.

Any attempt to propose a mission design minimizing the impact threat of a fragmented NEO will require a better understanding of the fragmentation process. It is hoped that current simulation of a NEO breaking up under a high energy deflection attempt will be able to identify the relationship between details of the fragmentation process and the initial conditions of an orbital dispersion simulation. The comparison of multiple initial parameters will require further research into how individual parameters affect the overall outcome. This can then allow for a weighting process to be designed, or for another form of multivariable optimization to be implemented.

A better description of the system dynamics should involve collisions between the NEO fragments. Since this process is not known deterministically, it introduces additional random factors into the model. These would include variable coefficients of restitution, inelastic collisions, additional fragmentation of particles, and the time scales for collision modeling. All of these factors should be analyzed and classified by the result of their variance on the expected outcome of the system.

There have been concerns about the accuracy of linearized models on long time scales. These equations assume a small variation in θ along the range of the computational model. With the expected increase in the transverse distribution of a debris cloud for the case of subsurface nuclear explosions with a large dispersal speed of 10 m/sec [14], compared to the nuclear standoff explosion case considered in this paper, the relative motion model may not preserve the distances needed for mutual gravitational acceleration terms. The shapes of relative ellipse contours need to be taken into account for large in-orbit motion described in [14], which is not embodied by Eq. (8). However, after a few orbits, the debris cloud will simply keep elongating along the orbital path.

Finally, while a large database of simulations has been compiled for this model, there is additional data analysis that must be done. The sample cases presented provide examples that validate the model and show expected outcomes, but work to better parallelize the model will hopefully allow for additional insights into the complicated problem of orbital dispersion for a fragmented NEO.

5. Conclusions

The expected outcome of short lead-time fragmentation of asteroid Apophis obtained using the approach presented in the paper agrees with previous study results. Fragmentation is most effective when the energy is imparted along the tangential direction of the NEO orbit, but it still results in less than 20% of the impact mass missing the Earth over 5 years. Therefore, isotropic breakup and dispersion of a NEO similar to Apophis with a short lead time is not a desired result. Missions for the deflection of these bodies should take this into account, and take measures to scale back the expected energy input or otherwise avoid the strong compression waves that may fragment the body.

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