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TARGET ASTEROID SELECTION FOR HUMAN EXPLORATION OF NEAR EARTH OBJECTS

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In recent years NASA has performed several studies to determine the feasibility of sending a crewed mission to an NEO using technology developed for under the Constellation project. A full search of the NEO database and accompanying mission analysis for each possible target asteroid is performed in this paper. The primary mission will be a short 90 day spaceflight. However, missions up to 180 days are also considered. Only asteroids with close flybys in the 2020-2030 ranges, corresponding to expected lunar mission dates, are considered.

INTRODUCTION

The Asteroid Deflection Research Center (ADRC) at Iowa State University has conducted a study to develop the software and procedures to automate the process of determining a list of possible target asteroids for a crewed mission to a near-Earth object (NEO). This study will outline launch windows and opportunities for 90-180 day crewed missions to asteroids. A list of possible asteroids will be determined using NASA's NEO list. Only asteroids mission in the 2020-2030 range will be examined. Several possible mission configurations capable of a crewed NEO mission will be discussed as well. Configurations ranging from the full constellation architecture to using EELV's will be considered.

NEO Background

Comets and asteroids that approach and/or cross Earth's orbit are known as near-Earth objects (NEOs). For this study no comets, approximately 10% of the NEO population, will be considered because of the low eccentricity and semi-major axis mission constraints. NEOs typically range in size from several meters to tens of kilometers. However, the smaller objects drastically outnumber the larger asteroids. Many NEO orbits are very similar to Earth's orbits, which often take less ΔV than a typical lunar mission, Apollo or planned Constellation mission. This is mostly because braking and departure ΔV's are often much lower than lunar missions.

In 1998 NASA accepted an mandate to detect and catalog 90% of NEOs larger than 1 km.[1,2] The 2005 NASA authorization act mandates NASA to detect and characterize NEOs down to 140 m in diameter. At the time of this publication there are 6600+ NEOs known with the rate of discovery predicted to vastly increase over the next decade.[1] It has been predicted that up to 90% of all NEOs with a diameter of 140 m or more will be detected by 2020.

With the list of asteroid constantly growing it is extremely important to develop a computer program, using MATLAB, capable of determining all the possible crewed NEO targets automatically with limited user input required. The purpose of this study was to develop such a search tool. The results from using the program will be presented throughout this paper, with some of the details of the computational algorithm presented through out the appendices.

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Rationale for Crewed NEO Mission

There are numerous benefits and rationale for sending a crewed mission to an NEO. Firstly an NEO mission is less risky than a lunar or Mars mission because a lander is not needed to get to an NEO. Also, it may be possible to utilize asteroids for their material resources. A crewed NEO mission could quickly validate or disprove this. An NEO mission would also help develop deep-space operational experience. This will be critical for a crewed Mars mission and any other mission aimed at building a human presence in the inner solar system, making NEOs a crucial stepping stone to Mars. Further detailed analysis could offer insight into how the solar system was formed. Finally, and perhaps most importantly, a crewed NEO mission would serve to inspire today's youth to pursue careers in science and engineering, much as the Apollo program did in the 60's and 70's. In addition having the capability to reach an NEO enables deflection missions, if ever necessary.

Past Crewed NEO Mission Studies

The idea of sending a crewed mission to an NEO has been proposed many times throughout the past few decades. In 1966, a study was performed at NASA that proposed sending a 500+ day crewed mission to the asteroid 433 Eros. The proposed mission would have taken place in 1975 when 433 Eros passed within 0.15 AU of the Earth.[3] Later, manned NEO studies performed by O'Leary[4] outlined the necessary mission requirements to mine several near-Earth and main-belt asteroids. The proposed missions required 1-3 year total mission times, well beyond the mission length of any recent manned NEO study. Several other studies were performed in the late 1980s as part of the Space Exploration Initiative, however interest in a manned NEO mission declined drastically until recent years.

Interest in crewed NEO missions has again begun to increase as NASA's Constellation Program matures. Over the past few years, several studies have been conducted to determine the feasibility of using Constellation hardware for a crewed NEO mission for a 90-180 day spaceflight with a 7-14 day asteroid stay times.[1,2,5] Also, in 2009 the Review of Human Spaceflight Plans Committee proposed replacing the current Constellation program with a flexible path option that would likely include a crewed NEO mission.[6] The recent NASA studies have searched for optimal launch dates for several asteroids for approximately 2015-2100. In this paper detailed mission designs will be discussed for crewed NEO missions only for 2020-2030, a much more likely date for the first crewed NEO mission, especially if the flexible path option is ultimately chosen. The following asteroid search is applicable regardless of the hardware that is available for the mission.

DETERMINING TARGET LIST

The first step in searching for targets for a crewed NEO mission is to obtain the full list of NEO, which can be found on NASA's Near Earth Object Program's website. The list used for this study contains 6646 asteroids. The first step is to narrow this list of asteroids before using the mission design software, developed by the Asteroid Deflection Research Center, to search for possible crewed NEO targets. With the current efficiency of the program a full search of the NEO database would take 3-4 weeks of computation time. More information and the developement of the program and its efficiency can be found in the appendices. Careful evaluation of previously designed crewed NEO missions reveals several properties and observations, listed below, that can be used to further narrow the 6600+ asteroid list.[5,7]

1. Only asteroids will be considered, no comets or extinct comets.
2. Minimum ΔV occurs near an Earth close approach. In this study and close approaches within 0.2 AU of the Earth were processed.
3. The asteroid must have a low eccentricity and inclination (Earth-like orbits), more to be discussed later.
4. In general, only asteroid with slow rotations that are single solitary objects should be considered. However, for this study neither of these properties were considered.

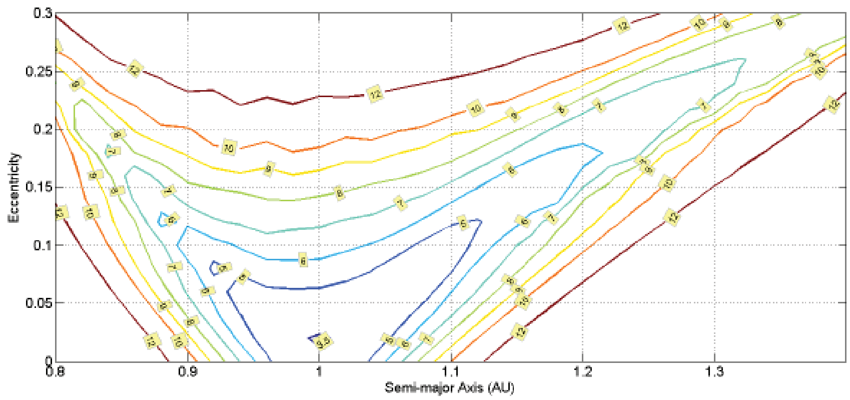

Figure 1: Total ΔV contour plot required to the early and late launches. This plot is for the 180-day mission.

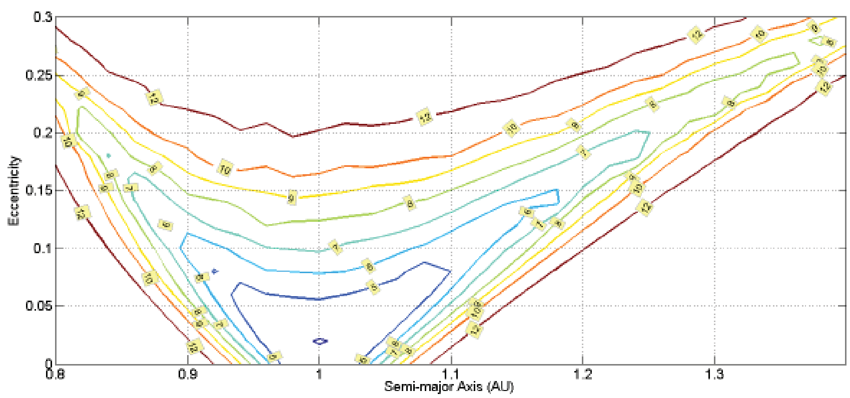

Figure 2: Total ΔV contour plot required to the early and late launches. This plot is for the 90-day mission.

Specific limits for the semi-major axis and eccentricity were obtained by developing a contour plot of the total ΔV for eccentricity versus semi-major axis. The 180-and 90-day contour plots can be seen in Figs. 1 and 2 respectively. The asteroid list was narrowed down using the semi-major axis and the eccentricity limits from 180-day contour plot. Many of the asteroids on this list have high ΔV's for mission lengths less than 180 days, however the list obtained from the 180-day semi-major axis and eccentricity limits was used for the 60, 90, 120, 180, and 360 days searches as well. In the recent NASA studies, the longest allowed mission length was 180 days. Using the Orion capsule, it is difficult to carry enough supplies for a 2 person crewed for longer missions. The 360-day case was run to show that there are 3+ times as many targets for a 360-day mission as there is for a 180-day mission. This final limits for the semi-major axis and eccentricity used in this study are given below.

- $0.8 \leq a \leq 1.325$ (AU)
- $0 \leq e \leq 0.26$

Between 2020 and 2030, there are approximately 1400 asteroids that have an Earth close approach within 0.2 AU, which was the close approach limit used. By then using the semi-major axis and eccentricity limits listed above, the list can be narrowed down further to 111 separate asteroids with a total of 169 close approaches that must each be checked. This list of asteroids was then processed to produce all the data and figures given in the following sections. All asteroids that fit the criterion with an inclination of less than 10^o were checked.

Mission Analysis

In general, two launch dates are found for each close approach that have similar ΔV's.[7] The first launch date occurs approximately the total mission length before the close approach, with the majority of orbital maneuvers performed within the last 2-3 weeks of the mission. With the majority of maneuvers, except the Earth departure burn, being conducted in the last 2-3 weeks of the mission, these missions may not be feasible unless the problem of storing cryogenic fuels for the long term is solved. The second launch date usually occurs within a few days of the close approach date. The asteroid arrival and departure burns are then performed within the first 2-3 weeks of operations. The majority of burn for the later launch date makes these launch date the better choice because cryogenic fuels will most likely be used (cryogenics are used in the Ares V EDS and the Centaur upper stage). Cryogenic fuels offer much better performance than bi-propellant fuels, so they will most likely be used for any crewed mission beyond low-Earth orbit. The use of cryogencic fuels and possible mission architecture configurations will be discussed later in the mission configuration section. These early and late launch windows can be seen in Fig. 3. Approximately 180 days prior to the April-13-2029 Earth-Apophis close encounter there is a launch window, as well one within a few days of the close encounter.

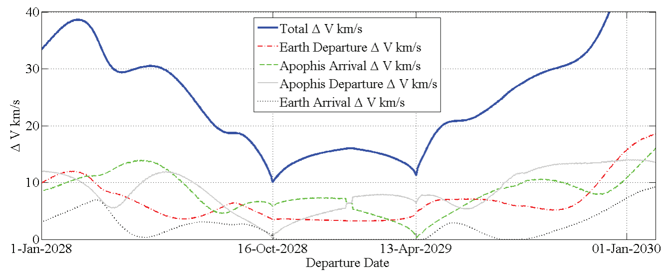

Figure 3: Plot of ΔV versus launch date for the 4/13/2029 Apophis close approach.

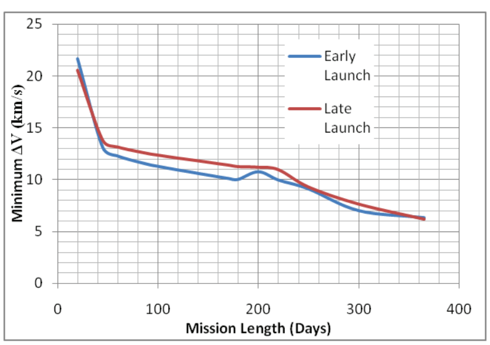

Figure 4: Plot of ΔV versus mission length for the 4/13/2029 Apophis close approach.

Another useful property that can be seen through careful examination of the mission design results is that in general, the longer the mission the lower the total ΔV is. This is true, especially for a crewed NEO return mission. This trend can be seen in Fig. 4, which is a plot of total ΔV versus mission length for both the early and late launch dates of the April-13-2029 close approach.[7] From Fig. 4 it can be seen that a 90-day mission to Apophis requires a minimum ΔV of approximately 11.5 km/s, while a 360-day mission requires approximately 6 km/s of ΔV. It is also worth mentioning that as the mission length changes, and with the required ΔV, changes the general shape of the minimum ΔV versus launch date stays the same. The total ΔV line moves upward and compacts toward the close approach date slightly as the mission length decreases. The opposite happens when the mission length is increased. The general trend is illustrated by the arrows in Fig. 5.

A computer program was written to search for the minimum ΔV for each of the 111 asteroid and 169 close approach dates from the list determined earlier. The majority of the software combines ephemeris data with a Lambert solver and various other functions to determine launch windows and trajectories for each of the asteroids. The only input required for the software is the desired mission length, each asteroid's ephemeris data, and a table downloaded from NASA of the entire NEO list and a list of the close encounters. When

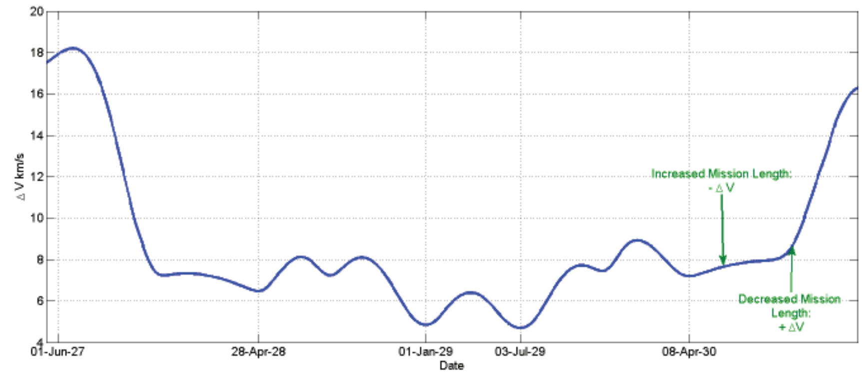

Figure 5: Plot of total mission ΔV versus launch date for a 180-day mission to 2000 SG344.

new NEOs are found, ranging from 100's, possibly 1000's every year, a new search can be performed and completed with a few hours. This software was designed to be able to easily run new searches when asteroids are added to the NEO list. Details on how the Lambert solver is used, how the ephemeris data is used, and other various functions required for the search can be found in Appendices A and B. A typical trajectory found by the program is shown in Fig. 6, which is the early launch date for 1999 AO10 close approach date on Feb. 2nd, 2026. This mission has a 180 day trajectory with a total ΔV of 6.7 km/s. Throughout this paper, the maximum cutoff ΔV used to create the possible asteroid lists is 7.2 km/s, which is the approximate maximum expected from Constellation hardware (Orion and Ares V). The launch opportunities and additional information from the program are presented in the following section.

Launch Opportunities

A list of all the feasible (7.2 km/s ΔV limit) NEOs for a 180-day mission is shown in Table 1. A majority of the asteroids with possible launch windows have small diameters ranging from approximately 4 to 43 meters, with diameters of asteroids 2009 OS5 and 2006 WB unknown at this time. A majority of the asteroid have very low inclinations, less than 3 degrees, with the exception of 2008 LD (6.5 deg), 2006 WB (4.9 deg), and 2007 XB23 (8.5 deg). The launch dates found range from late 2019 to early 2030.

The maximum mission length considered in this section is 180 days, corresponding to the maximum mission length proposed in the current NASA studies. A list of target asteroids for the 90-day, 120-day, 180-day cases is shown in Table 2. A total of 11 asteroids are found, with 16 separate launch windows for the 180-day mission. For both the 90-day and 120-day cases only two asteroids have been found, 2006 RH120 and 2000 SG344. In the 45-and 60-day cases, no asteroids were found to be reachable given all the studies limitations. The trajectory for the 180-day mission to 1999 AO10 can be seen in Fig. 6. More mission trajectories can be seen in Appendix D.

The 360-day launch windows and trajectories were processed using the list obtained from the 180-day limits as well. In this case, 45 of the 111 asteroids processed are feasible targets using the 7.2 km/s ΔV limit. This means that missions closer to a year in length are still achieveable if the Ares V or other heavy lifter is not available. A list of asteroid selected for the 360-day mission length is also provided in Appendix D. To shorten the list, only asteroids that require a ΔV of approximately 5 km/s are shown.

LAUNCH VEHICLE CONFIGURATIONS

With a list of asteroid and the required minimum ΔV's calculated, the next step is to give a quick outline of possible mission architecture configurations. The recent crewed NEO studies[2,5,1] from NASA have focused on how the Constellation hardware can be utilized for such a mission. Given current economic and political

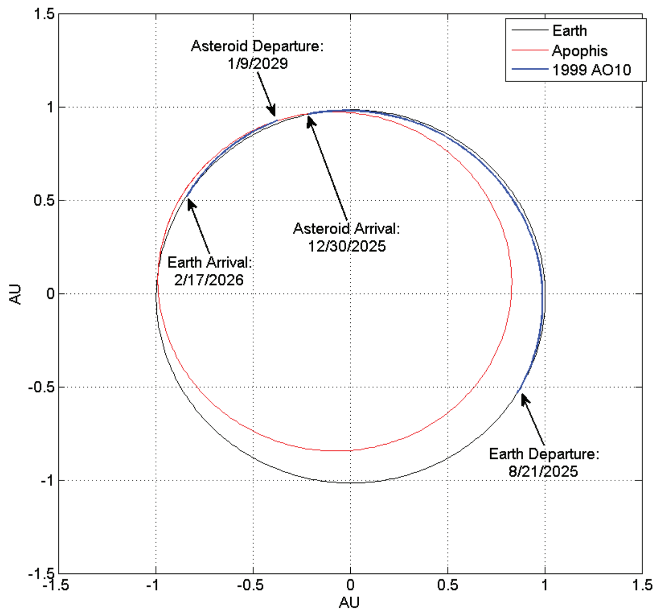

Figure 6: 180 trajectory to asteroid 1999 AO10 departing Eart in August 2025. This launch is approximately 180 days prior to the close encounter

Table 1: Orbital and physical parameters for 11 target asteroid found for the 180-day mission.

Asteroid	Close Approach Date	Semi-Major Axis(AU)	Eccentricity	Inclination (deg)	Diameter (km)	H
2008EA9	25-Apr-2020	1.059	0.080	0.424	0.01	27.7
2008LD	28-May-2020	0.892	0.155	6.542	0.006	28.9
2009OS5	14-Jul-2020	1.144	0.097	1.695	-	23.7
2001GP2	3-Oct-2020	1.038	0.074	1.279	0.014	26.9
2006WB	26-Nov-2024	0.850	0.181	4.909	-	22.8
2007XB23	11-Dec-2024	1.041	0.054	8.530	0.013	27.1
2008JL24	16-Oct-2025	1.038	0.107	0.550	0.004	29.6
1999AO10	11-Feb-2026	0.912	0.111	2.622	.05-.07	24.8
2003LN6	19-Jun-2026	0.857	0.211	0.633	0.043	24.5
2000SG344	7-May-2028	0.977	0.067	0.111	0.037	24.8
2006RH120	8-Aug-2028	1.033	0.024	0.596	0.004	29.6

situations, the Constellation program may be canceled permanently. Given this situation, mission configurations that don't rely solely on Constellation hardware need to be considered. Without Constellation's Ares

Table 2: Launch dates with corresponding asteroids and total mission ΔV for 180-day, 120-day, and 90-day mission lengths.

Asteroid	Earth Departure Date	Min ΔV (km/s)
180 Day Mission		
2007 XB23	11-Dec-24	4.525
2006 RH120	16-Jun-28	4.655
2000 SG344	3-Jul-29	4.695
2000 SG344	29-Jan-29	4.842
2000 SG344	8-Apr-2030	7.198
2007 XB23	14-Jun-24	5.365
2008 EA9	21-Nov-19	5.386
2008 JL24	15-Sep-25	6.169
2008 JL24	7-Feb-26	6.331
2000 SG344	28-Apr-28	6.473
1999 AO10	21-Aug-25	6.704
2009 OS5	14-Feb-20	6.842
2001 GP2	7-Apr-20	6.913
2003 LN6	21-Dec-25	6.975
2006 WB	31-May-24	7.014
2008 LD	27-May-24	7.063
120 Day Mission		
2006 RH120	10-Jul-2028	5.24
2000 SG344	28-Jul-2029	5.78
90 Day Mission		
2006 RH120	21-Jul-2028	5.93
2000 SG344	27-Jul-2029	7.19

V or similar heavy lift capability, the required maximum ΔV will be significantly lowered. For this reason a crewed NEO mission without heavy lift capability will have to be significantly longer. However, mission requiring less than 4 km/s of ΔV are possible.

Constellation Overview

NASA's Constellation program is comprised of the Orion CEV, Ares I, Ares V, and the Altair Lunar Lander.[8] The Orion CEV is to be launched using the Ares I, making this combination essentially the replacement for the retiring shuttle. For a lunar mission, the Ares V will then be used to launch the Altair Lunar Lander, along with fuel contained in the Earth Departure Stage (EDS) of the Ares V. The Orion would then rendezvous in orbit with the Altair and EDS prior to departing for the moon. The Orion and Ares I are to be built first, followed by the Ares V, and finally the lunar lander. There will be a short period of time where the design and construction of the Ares V is completed before the lander is completed. During this window of time it would make sense to launch a crewed NEO mission, because the lander is not required. In most cases, landing on the asteroid may not be possible because of their small size and low gravity.

The Ares V is currently planned to have the capability to launch approximately 187.7 metric tons into low earth orbit, but is not intended to be man rated. The current crewed NEO studies all assume the Ares V will be man rated and capable of launching the Orion. This is the case where the 7.2 km/s ΔV previously mentioned was derived. In the following section, several possible mission architectures will be discussed using Constellation and non-Constellation technology.

Mission Configurations

In this section, it will be shown that it may be possible to launch a crewed NEO mission using Constellation hardware as well as mission configurations that don't require the use of Constellation hardware. Table 3 is a list of all the items used in this trade-off study, as well as the dry and propellant masses and the specific impulse of each item.[8] The information for the Dragon (SpaceX's planned crewed capsule) are rough estimates of what the craft may be like. The Soyuz capsule is in the same mass and size class as the proposed Dragon, so it may be possible to utilize a modified Soyuz capsule as well. In addition, a Dragon class of crewed CEV may need to be further reinforced to withstand the radiation levels that occur beyond LEO. In either case the crewed vehicle will likely have to be modified to provide additional life support and radiation shielding for an extended mission. Additional ΔV may also be required at Earth arrival because the entry velocities were usually in the 11 km/s range, most likely too high for the Dragon or Soyuz. These issues are well beyond the scope of of this paper and will not be further mentioned.

Table 3: Items used for mission configuration tradeoffs.[7,8]

	Total Mass (kg)	**Burnout Mass (kg)**	**Fuel Mass (kg)**	**I_{SP} (sec)**
Orion	20500	11204	9297	323
Dragon Class	6000	4700	1300	320
Altair Lunar Lander	43000	20000	23000	450
EDS (Pre TLI Burn with Altair)	120496	26600	93896	448
EDS (Pre TLI Burn with Orion)	142995	26600	116395	448
EDS (Fuel)	163496	26600	136896	448
Centaur	22743	1914	20829	451

Using Table 3, basic performances for various mission configurations can be analyzed using the standard rocket equation.[9] The ΔV capabilities of each configuration are listed in Table 4. The best performing system uses the Ares V, launching only the EDS with fuel, and the Ares I/Orion configuration, and is capable of approximately 7.67 km/s of ΔV. This configuration requires 2 launches and is not likely to be used for a crewed NEO mission (most likely budget limitations). The next best performing system uses the Ares V to launch the Orion capsule and is capable of approximately 7.21 km/s ΔV. This configuration assumes a man-rated Ares V as well.

Non-Constellation hardware crewed NEO missions may also be possible. A configuration utilizing a crewed vehicle in the Dragon class of CEV's and the Centaur upper stage may be capable of over 6 km/s of ΔV. The Centaur can be launched using an EELV, either the Atlas V or Delta IV. With a slightly lower ΔV than the Constellation derived configurations the mission length will need to be expanded beyond the 180 day maximum limit of the recent crewed NEO studies.

Table 4: Performance characteristics for various mission architectures.

Configuration	Total $\triangle$V (km/s)
Ares I, Orion + Ares V, Altair	6.37
Ares V, Orion	7.21
Ares 1, Orion + Ares V(fuel)	7.67
Ares I, Orion + EELV, Centaur	4.57
Dragon Class +EELV, Centaur	6.0+

CONCLUSION

A software package has been developed that will allow for future launch window and trajectory searches as additional NEOs discovered. Using this software, approximately 11 target asteroids, for the 2020-2030 ranges, have been determined for a 180-day crewed NEO mission. Launch windows and trajectories for 90-day, 120-day, and 360-day missions have also been obtained. Details on solvers and functions used in the program can be found in the appendices. Lastly, it has been shown that a crewed NEO mission may be feasible with or without Constellation hardware.

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APPENDIX A: SOLUTION TO LAMBERT'S PROBLEM

The purpose of the appendices are to lay the ground work necessary to build a computational tool to determine the trajectories and information presented in this paper. Several solutions to Lambert's problem have been tested to determine the most efficient solution (Appendix A). The transformation from orbital elements to state vectors will be covered as well as the inverse transformation from the state vectors to the orbital elements. Ephemeris data will also be needed to calculate the positions of the Earth and each asteroid that is tested. A linear solution to determine Earth's orbital elements[9] will be covered in Appendix B. Ephemeris data for each asteroid tested was obtained using NASA's HORIZONS System. A solution to Kepler's time equations using a Newton method is included in Appendix B as well.

For any given two position vectors and the time-of-flight (TOF), the orbit determination problem is called Lambert's problem. For this reason Lambert's problem is well suited for an initial orbit-determination and searching technique. In Lambert's problem the two position vectors and the TOF are known, but the orbit connecting those two points is unknown. The classical Kepler problem is used to determine a position as a function of time where the initial position and velocity vectors are known. Various solutions so Lambert's problem can be found in the literature.[9,10,11,12] While many solutions methods have been proposed, two commonly used methods are employed, namely the classical universal variable method[9,11,12] and Battin's more recent approach using an alternate geometric transformation than Gauss' original method.[10,11] The search for launch dates often requires Lambert's problem to be solved thousands, often millions of time. For this reason the object of this section is to determine the most efficient method to solve Lambert's problem.

Lambert Problem Definition

In Lambert's problem the initial and final radius vectors and time-of-flight are given respectively as, $\vec{r}_0$, $\vec{r}$, and Δt. The magnitudes of $\vec{r}_0$ and $\vec{r}$ are defined as r_0 and r. The following parameters are also needed for the post processing for each solution method.

$$c = \mid \vec{r} - \vec{r}_0 \mid \tag{1}$$

$$s = \frac{r_0 + r + c}{2} \tag{2}$$

A method to determine the transfer angle $\Delta\theta$ without quadrant ambiguity is described below.[9] The transfer angle $\Delta\theta$ for a prograde orbit is determined as follows:

$$\Delta\theta = \begin{cases} \cos^{-1}\left(\dfrac{\vec{r}_0 \cdot \vec{r}}{r r_0}\right) & \text{if } (\vec{r}_0 \times \vec{r}) \geq 0 \\ 360^o - \cos^{-1}\left(\dfrac{\vec{r}_0 \cdot \vec{r}}{r r_0}\right) & \text{if } (\vec{r}_0 \times \vec{r}) < 0 \end{cases} \tag{3}$$

Similarly the transfer angle for a retrograde orbit is determined as:

$$\Delta\theta = \begin{cases} \cos^{-1}\left(\dfrac{\vec{r}_0 \cdot \vec{r}}{r r_0}\right) & \text{if } (\vec{r}_0 \times \vec{r}) < 0 \\ 360^o - \cos^{-1}\left(\dfrac{\vec{r}_0 \cdot \vec{r}}{r r_0}\right) & \text{if } (\vec{r}_0 \times \vec{r}) \geq 0 \end{cases} \tag{4}$$

Battin's Solution

Let's consider Battin's approach first proposed in the 1980's and later published in his book.[10] This solution is similar to the method used by Gauss, but moves the singularity from 180^o to 360^o and dramatically improves convergence when the θ is large. It should also be noted that Battin's solution works for all types of orbits (elliptical, parabolic, and hyperbolic) just as Gauss's original solution does. Throughout this section a brief outline of Battin's solution will be given. Any details left out can be found in Battin's book.[10]

Geometric Transformation of Orbit Lambert's problem states that the time-of-flight is a function of a, $r_0 + r$, and c. Therefore the orbit can be transformed to any shape desired as long as the semi-major axis a, $r_0 + r$, and c are held constant. For Battin's formulation[10,13,14] the orbit is transformed such that the semi-major axis is perpendicular to the line from $\vec{r}_0$ to $\vec{r}$, which is c by definition. The geometry of the transformed ellipse is shown in Battin's book.[10] From here the equation for the pericenter radius, r_p, is given below. The details leading up to this function can be found in Ref. (10).

$$r_p = a(1 - e_0) = r_{0p} \sec^2 \frac{1}{4}(E - E_0) \tag{5}$$

where e_0 is the eccentricity of the transformed orbit, while r_{0p} is the mean point of the parabolic orbit from P_0 to P.[10] The mean point of the parabolic radius r_{0p} is also given by:[10]

$$r_{0p} = \sqrt{r_0 r}\left[\cos^2\left(\frac{\Delta\theta}{4}\right) + \tan^2(2w)\right] \tag{6}$$

The value of $\tan^2(2w)$ is needed for the calculation, so w itself will not be defined. Next Kepler's time of flight equation can be transformed into a cubic equation, which must then be solved. First y will be defined as:

$$y^2 \equiv \frac{m}{(\ell + x)(1 + x)} \tag{7}$$

Kepler's time-of-flight equation can now be represented by the following cubic equation.

$$y^3 - y^2 - \frac{m}{2x}\left(\frac{\tan^{-1}\sqrt{x}}{\sqrt{x}} - \frac{1}{1+x}\right) = 0 \tag{8}$$

In the above equation l, m, and x are always positive and are defined below. To calculate l, the following two equations, dependent only on the geometry of the problem, are also necessary.

$$\epsilon = \frac{r - r_0}{r_0} \tag{9}$$

$$\tan^2(2w) = \frac{\frac{\epsilon^2}{4}}{\sqrt{\frac{r}{r_0}} + \frac{r}{r_0}\left(2 + \sqrt{\frac{r}{r_0}}\right)} \tag{10}$$

Using Eqs. (9) and (10), ℓ can be calculated as follows.

$$\ell = \frac{\sin^2\left(\frac{\Delta\theta}{4}\right) + \tan^2(2w)}{\sin^2\left(\frac{\Delta\theta}{4}\right) + \tan^2(2w) + \cos\left(\frac{\Delta\theta}{2}\right)} \qquad 0^o < \Delta\theta \leq 180^o \tag{11}$$

$$\ell = \frac{\cos^2\left(\frac{\Delta\theta}{4}\right) + \tan^2(2w) - \cos\left(\frac{\Delta\theta}{2}\right)}{\cos^2\left(\frac{\Delta\theta}{4}\right) + \tan^2(2w)} \qquad 180^o < \Delta \leq< 360^o \tag{12}$$

$$m = \frac{\mu \Delta t^2}{8 r_{op}^3} \tag{13}$$

$$x = \sqrt{\left(\frac{1-\ell}{2}\right)^2 + \frac{m}{y^2}} - \frac{1+\ell}{2} \tag{14}$$

Initial conditions for x that guarantee convergence are:

$$x_0 = \begin{cases} 0 & \text{parabola, hyperbola} \\ \ell & \text{ellipse} \end{cases} \tag{15}$$

The cubic function (Eq. (8)) can now be solved using the following sequential substitution method:

1. An initial estimation of x is given by Eq. (15).
2. Calculate all the values needed for the cubic from Eqs. (9), (10), (11) or (12), and (13).
3. Solve the cubic Eq. (8) for y.
4. Use Eq. (14) to determine a new value for x.
5. Repeat the above 3 steps until x stops changing.

We now have a nearly complete solution algorithm for Lambert's problem, although solving the cubic function is not a trivial matter. Battin[10] next develops a method to flatten the cubic function to improve the convergence rate, as well as a method using continued fractions to determine the largest real positive root of the cubic function. The following section gives the equations necessary to find the flattening parameters and solve the cubic function. Again, a much more detailed derivation can be found in Ref. (10).

Use of Free Parameter to Flatten Cubic Function The cubic equation, Eq. (8), from the previous section can now be represented in terms of the flattening parameter functions, h_1 and h_2, as follows:

$$y^3 - (1 + h_1)y^2 - h_2 = 0 \tag{16}$$

The flattening parameters can be represented in terms of x, l, and m as

$$h_1 = \frac{(l + x)^2(1 + 3x + \xi)}{(1 + 2x + l)[4x + \xi(3 + x)]} \tag{17}$$

$$h_2 = \frac{m(x - l + \xi)}{(1 + 2x + l)[4x + \xi(3 + x)]} \tag{18}$$

The function $\xi(x)$ needed for the calculation of h_1 and h_2 is calculated from the continued fraction as follows:

$$\xi(x) = \cfrac{8(\sqrt{1+x}+1)}{3 + \cfrac{1}{5 + \eta + \cfrac{\frac{9}{7}\eta}{1 + \cfrac{\frac{16}{63}\eta}{1 + \cfrac{\frac{25}{99}\eta}{1 + \cfrac{\frac{36}{143}\eta}{1 + \ddots}}}}}} \tag{19}$$

where η is defined as

$$\eta = \frac{x}{(\sqrt{1+x}+1)^2} \qquad \text{where} \qquad -1 < \eta < 1 \tag{20}$$

Solving the Cubic Function In this section a method to determine the largest real root of Eq. (16) is outlined. Using this method a successive algorithm can be used ultimately to determine the f and g functions, given by Ref. (11,12). Using the Lagrangian f and g functions the initial and final velocity vectors are then obtained. The first step is to calculate B and u as follows:

$$B = \frac{27h_2}{4(1 + h_1)^3} \tag{21}$$

$$u = \frac{B}{2(\sqrt{1+B}+1)} \tag{22}$$

Also, $K(u)$ is the calculated from the continued fraction

$$K(u) = \cfrac{\frac{1}{3}}{1 + \cfrac{\frac{4}{27}u}{1 + \cfrac{\frac{8}{27}u}{1 + \cfrac{\frac{2}{9}u}{1 + \cfrac{\frac{22}{81}u}{1 + \ddots}}}}} \tag{23}$$

where the coefficients for the odd and even coefficients of u are generally obtained from the following two equations.

$$\gamma_{2n+1} = \frac{2(3n+2)(6n+1)}{9(4n+1)(4n+3)} \tag{24}$$

$$\gamma_{2n} = \frac{2(3n+1)(6n-1)}{9(4n-1)(4n+1)} \tag{25}$$

The largest positive real root for the cubic equation is calculated as

$$y = \frac{1+h_1}{3}\left(2 + \frac{\sqrt{1+B}}{1+2u(K^2(u))}\right) \tag{26}$$

The cubic function (Eq. (16)) can now be solved using the following sequential substitution method.

1. An initial estimation of x is given by Eq. (15).
2. Calculate all the values needed for the flattening parameters Eqs.(17) and (18) from Eqs. (9), (10), (11) or (12), and (13).
3. Calculate $\eta(x)$ from Eqs. (19) and (20).
4. Calculate $K(u)$ using Eqs. (21), (22), and (23)
5. Calculate the solution for the cubic function using Eq. (26) for y.
6. Use Eq. (14) to determine a new value for x.
7. Repeat the above 5 steps until x stops changing.

The next step is to determine the semi-major axis of the orbit. If the semi-major axis is positive,the orbit is elliptical, and the initial and final velocity vectors can be calculated as follows. The hyperbolic and parabolic velocity vectors are calculated in a similar manner.[11]

$$a = \frac{\mu(\Delta t)^2}{16 r_{op}^2 x y^2} \tag{27}$$

Determining the Lagrange Coefficients for Elliptical Orbits The Lagrange coefficients for the parabolic orbit case can be calculated from the following set of equations.

$$\beta_e = 2\sin^{-1}\sqrt{\frac{s-c}{2a}} \tag{28}$$

$$\beta_e = -\beta_e \quad \text{If } \Delta\theta > \pi \tag{29}$$

$$a_{min} = \frac{s}{2} \tag{30}$$

$$t_{min} = \sqrt{\frac{a_{min}^3}{\mu}}(\pi - \beta_e + \sin\beta_e) \tag{31}$$

$$\alpha_e = 2\sin^{-1}\sqrt{\frac{s}{2a}} \tag{32}$$

$$\alpha_e = 2\pi - \alpha_e \quad \text{If } \Delta t > t_{min} \tag{33}$$

$$\Delta E = \alpha_e - \beta_e \tag{34}$$

The Lagrangian coefficients can be calculated as follows:

$$f = 1 - \frac{a}{r_0}(1 - \cos\Delta E) \tag{35}$$

$$g = \Delta t - \sqrt{\frac{a^3}{\mu}}(\Delta E - \sin\Delta E) \tag{36}$$

$$\dot{g} = 1 - \frac{a}{r}(1 - \cos\Delta E) \tag{37}$$

With the Lagrangian coefficients now known, the initial and final velocity vectors can be found from the following equations.

$$\vec{v}_0 = \frac{\vec{r} - f\vec{r}_o}{g} \tag{38}$$

$$\vec{v} = \frac{\dot{r}\vec{r} - \vec{r}_o}{g} \tag{39}$$

The f and g functions can also be determined for the hyperbolic case.[11]

This concludes the solutions to Lambert's problem using Battin's method.

Computational Efficiency for Each Solution Method

In this section an overview of the performance obtained from various Lambert problem solutions will be discussed. The three methods covered in this section are Battin's solution (discussed above), a universal variable using a Newton method, similar to BMW[12] and Curtiss,[9] and lastly a universal variable solution using the bisection method, similar to Vallado.[11]

Table 5: Information on the computer used for the computational efficiency study.

Model	Dell T3500 Workstation
Operating System	Windows Vista Enterprise 64 bit
Processor	2 x Intel(R) Xeon(R) W3520 2.67 GHz
Memory	6.00 GB 1066 Mhz DDR3
Matlab	R2009b-64 bit

For all calculations Matlab R2009b (64 bit version) was used. A compile environment would likely increase the general speed of the program, however the program is being written so the final version can be easily used and modified by others at the ADRC. Table 5 has all the information relevant to the computer used for these calculations. The computer used was a dual quad core Xeon processor (8 total cores) running Windows Vista Enterprise.

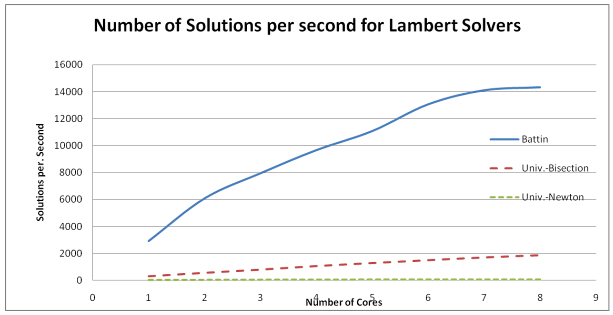


Figure 7: Plot of number of solutions per second versus number of cores for various Lambert solution methods.

In each case the program was ran for an example case that is representative of cases that the program would be likely to be used for. In this case it was ran to compute a porkchop plot for and asteroid rendezvous mission. A plot of the number of solutions per second for each of the three solutions is given in Fig. 7. Analysis of this plot indicates that the Battin solution is significantly more efficient than either the Newton or bisection solutions. In the same way the Newton universal variable solution is much slower than the bisection method. In general the Newton method would be expected to be more efficient than a bisection method. In this case it is likely that the Newton method doesn't always converge and runs through every loop iteration allowed by the program. On average the Battin Solution is 8.94 times faster than the bisection method and 158.66 times faster than Newton solution method. The maximum number of solutions obtain using all 8 cores (maximum in every case) for the Battin, Newton, and bisection method are 14317, 1867, and 174 respectively.

APPENDIX B: MAIN FUNCTIONS FOR THE SEARCH PROGRAM

Ephemeris Data

To calculate the ephemeris data the Julian date must first be calculated.[9] Once the Julian date is determined, the planetary orbital elements can be calculated from ephemeris tables. The method presented in this section has linear drift rates for each orbital elements, as well as initial values. In this way the orbital elements of each planet can be determined as a function of Julian date. Table (6) shows the drift rates and initial values for Earth.[9] A table with all the information for the other 7 planets and Pluto can be found in Ref. (9).

Table 6: Table of Earths orbital elements and drift rates used to generate ephemeris data.

a, AU $\dot{a}$, AU/Cy*	e $\dot{e}$, 1/Cy	i, deg $\dot{i}$,″†/Cy	Ω , deg $\dot{\Omega}$, ″/Cy	$\tilde{\omega}$, deg $\dot{\tilde{\omega}}$, ″/Cy	L, deg $\dot{L}$, ″/Cy
1.00000011 -0.00000005	0.01671022 -0.00003804	0.00005 -46.94	-11.26064 -18228.25	102.94719 1198.28	100.46435 129597740.6

* where Cy is a century, which is 36525 days.
† where ″ is symbol for an arcsecond, which 1/3600th of a degree.

The orbital elements are calculated in the following manner. If X is defined to be any of the 6 orbital elements, then the orbital elements can be calculated in the following manner.

$$X = X_0 + \dot{X}T_0 \tag{40}$$

where T_0, the number of Julian centuries between J2000 and the date, is calculated from the Julian date.

$$T_0 = \frac{JD - 2451545}{36525} \tag{41}$$

$$\omega = \tilde{\omega} - \Omega \tag{42}$$

$$M = L - \tilde{\omega} \tag{43}$$

From the mean anomaly (M) and the eccentricity (e), the eccentric anomaly (E) can be found. Then, the true anomaly (θ) easily can be found. Finding the eccentric angle from M and e requires the Newton iteration method to solve Kepler's time equation, which is described in Curtis.[9]

Obtaining the State Vector from Orbital Elements

With all six orbital elements found from the ephemeris routine, the state vector can be calculated. The radius vector is needed for the inputs to Lambert's problem, as well for plotting trajectories. The velocity vectors are also needed to calculated ΔV's when the each Lambert solution is obtained. The function resulting from this section will be useful in many other situations as well. The first step is to calculate the radius and velocity vectors in the perifocal frame.[9,11]

$$\vec{r}_{pqw} = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}\left\{\begin{array}{l}\cos\theta\\ \sin\theta\\ 0\end{array}\right\} \tag{44}$$

$$\vec{v}_{pqw} = \frac{h}{\mu}\left\{\begin{array}{l}-\sin\theta\\ e+\cos\theta\\ 0\end{array}\right\} \tag{45}$$

Where the specific angular moment, h, is given by

$$h = \sqrt{a(1-e^2)\mu} \tag{46}$$

The transformation matrix from the perifocal frame to the body centered frame is given by.[15]

$$Q_{pqw\rightarrow ijk} = \left[\begin{array}{ccc}\cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i & \sin\Omega\sin i\\ \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i & -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i & -\cos\Omega\sin i\\ \sin w\sin i & \cos w\sin i & \cos i\end{array}\right] \tag{47}$$

The radius and velocity vectors in the body centered frame are then transformed by

$$\vec{r}_{ijk} = Q\vec{r}_{pqw} \tag{48}$$

$$\vec{v}_{ijk} = Q\vec{v}_{pqw} \tag{49}$$

Obtaining Orbital Elements from State Vector

In this section a method to obtain the six classical orbital elements, a, e, i, Ω , ω, and θ, from the state vector will be discussed. The sixth orbital element is actually, t_p, but this can be obtained directly from θ. This conversion is necessary to propagate orbits and determine the orbital elements after the solution to Lambert's problem has been obtained. The first step is to determine the specific angular momentum vector as follows:

$$r = |\vec{r}| \tag{50}$$

$$v = |\vec{v}| \tag{51}$$

$$\vec{h} = \vec{r} \times \vec{v} \tag{52}$$

$$h = |\vec{h}| \tag{53}$$

The eccentricity vector and orbit eccentricity can be found as

$$\vec{e} = \frac{1}{\mu}\left[\vec{v} \times \vec{h} - \mu\frac{\vec{r}}{r}\right] \tag{54}$$

$$e = |\vec{e}| \tag{55}$$

With the angular momentum and eccentricity now known, the orbit's semi-major axis can be found as

$$a = \frac{h^2}{\mu}\frac{1}{1-e^2} \tag{56}$$

The inclination of the orbit can be obtained from the angular momentum vector as the inverse cosine of the z component of the angular momentum vector divided by the magnitude of the angular moment. By definition the inclination must be between 0^o to 360^o, so there won't be any quadrant ambiguities.

$$i = \cos^{-1}\left(\frac{h_z}{h}\right) \tag{57}$$

The node line vector is required to obtain both the right ascension of the ascending node, and the argument of perigee of the orbit. The node line is calculated from

$$\vec{N} = \vec{K} \times \vec{h} \tag{58}$$

$$N = |\vec{N}| \tag{59}$$

The longitude of the ascending node Ω is found from $\Omega = cos^{-1}\frac{N_x}{N}$, with the proper quadrant, as

$$\Omega = \begin{cases} \cos^{-1}\left(\frac{N_x}{N}\right) & N_y \geq 0 \\ 360^o - \cos^{-1}\left(\frac{N_x}{N}\right) & N_y < 0 \end{cases} \tag{60}$$

The argument of perigee and the true anomaly can be found similarly as

$$\omega = \begin{cases} \cos^{-1}\left(\frac{\vec{N}\cdot\vec{e}}{Ne}\right) & e_z \geq 0 \\ 360^o - \cos^{-1}\left(\frac{\vec{N}\cdot\vec{e}}{Ne}\right) & e_z < 0 \end{cases} \tag{61}$$

$$\theta = \begin{cases} \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}}{er}\right) & \vec{r}\cdot\vec{v} \geq 0 \\ 360^o - \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}}{er}\right) & \vec{r}\cdot\vec{v} < 0 \end{cases} \tag{62}$$

APPENDIX C: ALGORITHM USED TO CONSTRUCT CONTOUR PLOT

The contour plot shown in Figs. 1 and 2 can be obtained using the method outlined in this section. For both contour plots the inclination (i) and longitude of the ascending node (Ω) were set to zero. The minimum ΔV for each set of semi-major axis (a) and eccentricity (e) can be calculated as follows.

It was previously noted that the minimum total ΔV for a crewed NEO mission always occurs near a close encounter and that the minimum ΔV occurs with a launch near the close approach, and near the mission length before the close approach. At first glance it appears that the true anomaly, which is a function of the orbit's radius, semi-major axis, and eccentricity, can be solved by the following two equations.

$$p = a(1 - e^2) \tag{63}$$

$$\cos\theta = \frac{p - r}{re} \tag{64}$$

The asteroid's orbital radius is assumed to be equal to the orbit radius of the earth (1 AU) near the close approach(s). When the eccentricity is near zero, Eq. (64) will yield incorrect results, so the close approach true anomaly must be found numerically. This is done by choosing an argument of perihelion (ω) and checking the radius for various close approach angles. The radius for each of these combinations is calculated using the orbital element to state vector function described in Appendix B. The angle which gives the radius closest to 1 AU is then used at the true anomaly of the close approach. For each orbit two close approach can be found, the first between 0 and 180 degrees, and the second is between 180 and 360 degrees.

The next step in the process is to determine the close approach date for both the upper and lower true anomaly angles. This was done by finding the distance between the close approach radius vectors and Earth's radius vector, which was calculated for a 40 year period using 1 day time steps.

With the close approach date determined for both the true anomaly angles the next step is to determine the argument of perihelion, which results in the lowest total ΔV. There is no direct solutions for this step, so a numerical method was developed. Various values for the argument of perihelion from 0 to 360 degrees are then chosen and the minimum total ΔV for each is calculated.

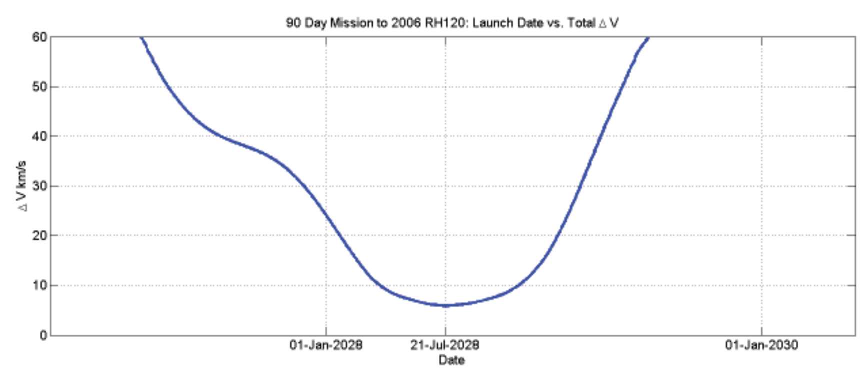
The minimum total ΔV is found by generating a pseudo ephemeris file of the proposed asteroid, using Kepler's equation to propagate the orbit. Using the Lambert solver described in Appendix A, the total ΔV for a return mission to the pseudo asteroid can be obtained using a program similar to the one used to determine the minimum ΔV for the crewed asteroid mission. A pseudo code to generate the ΔV matrix used for the contour plot is outlined below.

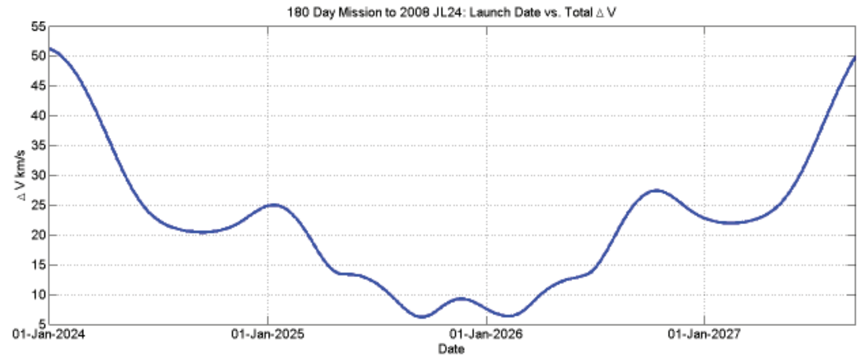
1. $\Omega = 0$ and $i = 0$
2. Loop 1: vary a
    3. Loop 2: vary e
    4. Determine θ_l and θ_u, the two true anomaly angles of the close approaches.
    5. Determine the two Earth close approach dates.
        6. Loop 3: vary ω
        7. Use the return mission program to determine the minimum total ΔV for a, e, i, Ω, ω, and θ.
        8. End
    9. Determine the minimum total ΔV for each a and e from each minimum found in loop 3
    10. End
11. End
12. Post processing done here.

It should be noted that this method is in no way optimized for optimal performance. The contour information only has to be calculated once, so as long as the calculations could be done in a reasonable amount of time (approximately 1-2 days). In this program, Lambert's problem is calculated tens of millions of times (only the Battin solver is feasible for this problem), depending on the mission length and accuracy desired. The 180-day contour plot took approximately 23 hours to run, and the 90-day contour plot took approximately 12 hours to run.

APPENDIX D: VARIOUS MISSION TRAJECTORIES AND DATA

Asteroid	Close Enounter Date	Min ΔV (km/s) 360 day mission
2000SG344	7-May-2028	3.61
2008UA202	18-May-2028	4.10
2006RH120	8-Aug-2028	4.25
2001GP2	3-Oct-2020	4.35
2009HC	26-Oct-2025	4.48
2007UN12	4-Jul-2020	4.60
2008EA9	25-Apr-2020	4.67
2006DQ14	16-Sep-2029	4.68
2007XB23	11-Dec-2024	4.74
2008JL24	16-Oct-2025	4.98
2006HE2	13-Apr-2028	5.04





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