

# GUIDANCE ALGORITHMS FOR ASTEROID INTERCEPT MISSIONS WITH PRECISION TARGETING REQUIREMENTS

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Rendezvous and intercept missions to smaller asteroids require precision guidance and control in the terminal mission phase. The zero-effort-miss (ZEM) and zero-effort-velocity (ZEV) information are used to formulate autonomous feedback guidance laws. A terminal-phase guidance strategy for a variety of intercept missions is developed. Different types of navigation information are assumed to be available from different spacecraft configurations and hardware choices. Guidance laws are developed for different mission configuration options. Guidance laws for both rendezvous and impact are studied. Optimal feedback guidance laws are also investigated for asteroid intercept/rendezvous missions. Simulations show the effectiveness of the various guidance laws.

## INTRODUCTION

Acknowledgment of the threat to planet Earth from the impact of an asteroid has led to increased interest in studying techniques to intercept and/or rendezvous with asteroids. In addition to responding to a threatening asteroid, scientific missions to visit asteroids, such as NASA's successful Deep Impact mission, will continue to drive interest in studying asteroid intercept and rendezvous. For many scenarios, it may be desirable to be able to command asteroid intercept at a specified approach angle or impact velocity. Other missions may require delivering a spacecraft to a point near the asteroid for proximity operations. Reliable guidance laws are required to achieve these goals.

In this paper, classical optimal feedback guidance theory is used, especially as it pertains to spacecraft guidance.<sup>1,2</sup> Classical proportional navigation (PN) guidance and its variants<sup>3</sup> are also considered. Optimal guidance laws based on the zero-effort-miss (ZEM) and zero-effort-velocity (ZEV) can command a spacecraft to a desired state, including both position and velocity, at the desired time. These laws can be made robust against disturbances using sliding-mode control implementation.<sup>4,5</sup>

Much work has been done on the problem of commanding intercept at a specified impact angle. Some of the earliest work led to guidance laws with strict limits on initial conditions.<sup>6</sup> Since then, a number of different laws with different advantages have been proposed. Guidance laws exist that do not require the time-to-go,<sup>7</sup> that allow for significant target maneuvers,<sup>8</sup> and that can be used during

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hypersonic flight.<sup>9</sup> Linear quadratic guidance laws have been developed,<sup>10</sup> as well as guidance laws based on classical proportional navigation.<sup>11</sup> Guidance algorithms have also been derived to follow a circular path to the target,<sup>12</sup> and to establish the desired end-of-mission geometry early on.<sup>13</sup> Many more impact-angle guidance concepts have also been developed by missile guidance researchers in the past.

The problem of intercepting a small solar system body at high impact velocities has been recently studied by spacecraft guidance researchers. NASA's Deep Impact mission successfully encountered comet Tempel 1.<sup>14</sup> Guidance algorithms to hit small, faint near-Earth objects (NEOs) have been tested with high-fidelity simulation software.<sup>15,16</sup> Recently, a range of impact missions, from hypervelocity (10 km/s) impact to low speed (100 m/s) intercept with requirements on the final impact angle has been investigated.<sup>17,18</sup>

There are several broad classes of targeting requirements imposed by the mission design. The simplest class is intercept, simply commanding the spacecraft to hit the target asteroid. This can be achieved by a variety of methods, including classical PN guidance, as well as predictive guidance schemes utilizing both orbital perturbation theory and statistical orbit determination.

The next class is intercept in a prescribed direction. A formulation using only the observed line-of-sight angle and line-of-sight rate can be used, however neither the time of flight nor the impact velocity can be specified. With knowledge of the states (position and velocity) of the target and the interceptor, impact can be achieved at a given angle and at a specified time. If desired, the impact velocity can also be specified.

In this paper, a variety of different missions are described and simulated. Guidance laws for missions to achieve impact at a specified time, with the possibility of achieving a specified approach angle or impact velocity are given. These guidance laws are simulated, and their performance is compared to other guidance laws. Limitations based on implementation requirements are discussed. In addition to impact, the case of guiding a spacecraft to a specified position and velocity near the asteroid for proximity operations is also simulated.

The guidance laws considered in this paper are optimal in a sense of minimizing the integral of acceleration squared. They are compared with other guidance laws. The guidance laws in this paper require more information input, but result in optimized laws.

Before continuing, a note on terminology is in order. In missile guidance, the impact angle is usually defined in terms of the inertial reference frame being used. The geometry of the target is not considered. For asteroid intercept, the angle of impact in the inertial frame is important for mission planning and operations. However, the angle that the spacecraft makes with the local horizontal of the asteroid's surface at impact is also important, whether for a subsurface penetrator or for pointing instruments. When considering the impact at the asteroid's surface, this angle is the impact angle, and the angle in inertial coordinates is called the approach (or intercept) angle. Because impact angle is common in missile guidance literature, and in this paper the local impact angle is not considered, the terms impact angle and approach angle will be used interchangeably.

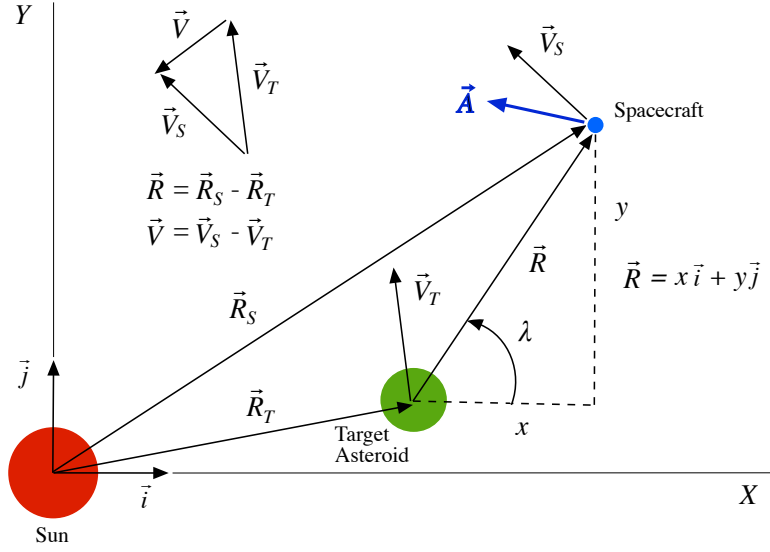


Figure 1. Two-dimensional intercept geometry.

## DYNAMICAL MODEL AND MISSION SCENARIO

### Dynamical Model

A target asteroid is modeled as a point mass in a heliocentric Keplerian orbit described by

$$\ddot{\vec{R}}_T = -\frac{\mu_{\odot}}{R_T^3} \vec{R}_T \quad (1)$$

where  $\vec{R}_T$  is the position vector of the target asteroid from the sun, and  $\mu_{\odot} = 1.32715 \times 10^{20} \text{ m}^3/\text{s}^2$  is the solar gravitational parameter. The heliocentric orbital motion of the interceptor spacecraft is described by

$$\ddot{\vec{R}}_S = -\frac{\mu_{\odot}}{R_S^3} \vec{R}_S - \frac{\mu_{\otimes}}{R^3} \vec{R} + \vec{A}_c \quad (2)$$

where  $\vec{R}_S$  is the position vector of the spacecraft from the sun,  $\mu_{\otimes}$  is the gravitational parameter of the target asteroid,  $\vec{A}_c$  is the control acceleration applied to the spacecraft, and  $\vec{R}$  is the position vector of the spacecraft from the target, defined as

$$\vec{R} = \vec{R}_S - \vec{R}_T \quad (3)$$

In this paper, asteroid Apophis with  $\mu_{\otimes} \simeq 15.35 \text{ m}^3/\text{s}^2$  is used as a reference target asteroid.

The equation of motion of the spacecraft with respect to the target becomes

$$\ddot{\vec{R}} = \vec{G} + \vec{A}_c \quad (4)$$

where  $\vec{G}$  represents the sum of apparent gravitational accelerations acting on the target, given as

$$\begin{aligned}
\vec{G} &= -\frac{\mu_{\odot}}{R_S^3} \vec{R}_S - \frac{\mu_{\otimes}}{R^3} \vec{R} + \frac{\mu_{\odot}}{R_T^3} \vec{R}_T \\
&= -\frac{\mu_{\odot}}{|\vec{R}_T + \vec{R}|^3} (\vec{R}_T + \vec{R}) - \frac{\mu_{\otimes}}{R^3} \vec{R} + \frac{\mu_{\odot}}{R_T^3} \vec{R}_T \\
&\cong -\frac{\mu_{\odot}}{R_T^3} \vec{R} + \frac{3\mu_{\odot}}{R_T^5} \vec{R}_T (\vec{R}_T \cdot \vec{R}) - \frac{\mu_{\otimes}}{R^3} \vec{R} \quad \text{for } R \ll R_T \\
&\cong -\frac{\mu_{\odot}}{R_T^3} \vec{R} + \frac{3\mu_{\odot}}{R_T^5} \vec{R}_T (\vec{R}_T \cdot \vec{R}) \quad \text{for } \mu_{\otimes}/R^3 \ll \mu_{\odot}/R_T^3
\end{aligned} \tag{5}$$

Note that  $\vec{R}_T$  can be assumed to be a known function of time and thus  $\vec{G} = \vec{G}(\vec{R}, t)$ .

From Figure 1 it can be seen that

$$\lambda = \arctan \frac{y}{x} \tag{6}$$

where  $\lambda$  is the line-of-sight (LOS) angle and  $(x, y)$  are the components of the relative position vector along the inertial  $(X, Y)$  coordinates. Differentiating this with respect to time gives

$$\dot{\lambda} = \frac{x\dot{y} - y\dot{x}}{R^2} \tag{7}$$

where  $\dot{\lambda}$  is the LOS rate and  $R = (x^2 + y^2)^{\frac{1}{2}}$ .

The closing velocity, the rate of change of the distance between the target and the spacecraft, can be found by differentiating  $R$  with respect to time as

$$V_c = -\dot{R} = \frac{-(x\dot{x} + y\dot{y})}{R} \tag{8}$$

## DERIVATION OF OPTIMAL FEEDBACK GUIDANCE LAWS

Three different optimal feedback guidance laws are considered for asteroid intercept and rendezvous. The various forms of proportional navigation and perturbation-based guidance laws compute an estimated mission time-to-go based on relative position and velocity, and use this computed time-to-go as an input to calculate the necessary acceleration commands. In contrast, the optimal feedback guidance laws studied in this paper use a specified time-to-go as a mission parameter, and compute the acceleration commands needed to achieve intercept at this pre-determined time.

When the final impact velocity vector (both impact velocity and impact angle) is specified, the terminal velocity is constrained. This leads to the constrained-terminal-velocity guidance (CTVG) law. If the final velocity is free, the free-terminal-velocity guidance (FTVG) law results. When only the approach angle is commanded, the velocity vector component along the desired final direction is free, while the perpendicular components are constrained to be zero. A combination of FTVG along the impact direction and CTVG along the perpendicular directions allows pointing of the final velocity vector, referred to as intercept-angle-control guidance (IACG).

### Constrained-Terminal-Velocity Guidance (CTVG)

The orbital equations of motion of a spacecraft are in general described by

$$\dot{\mathbf{r}} = \mathbf{v} \tag{9}$$

$$\dot{\mathbf{v}} = \mathbf{g}(\mathbf{r}, t) + \mathbf{a} \tag{10}$$

where  $\mathbf{r}$ ,  $\mathbf{v}$ ,  $\mathbf{g}$ , and  $\mathbf{a}$  are the position, velocity, gravitational acceleration, and control acceleration vectors, respectively. In general, we have  $\mathbf{g} = \mathbf{g}(\mathbf{r}, t)$ . However, depending on the problem, the gravitational acceleration can be assumed to be constant or negligible for practical applications. In this paper, we use the following standard vector notations:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}; \quad \mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Consider an optimal control problem for minimizing the integral of the acceleration squared, given as

$$J = \frac{1}{2} \int_0^{t_f} \mathbf{a}(t)^T \mathbf{a}(t) dt \quad (11)$$

subject to the equations of motion with specified initial and final conditions of  $\mathbf{r}$  and  $\mathbf{v}$ .

The Hamiltonian for this optimal control problem is

$$H = \frac{1}{2} \mathbf{a}^T \mathbf{a} + \mathbf{p}_r^T \mathbf{v} + \mathbf{p}_v^T (\mathbf{a} + \mathbf{g}) \quad (12)$$

Assuming that the gravitational acceleration  $\mathbf{g}$  is a constant vector, especially for a certain terminal-phase guidance problem, we obtain the costate equations and the optimal control condition as

$$\dot{\mathbf{p}}_r = - \frac{\partial H}{\partial \mathbf{r}} = 0 \quad (13)$$

$$\dot{\mathbf{p}}_v = - \frac{\partial H}{\partial \mathbf{v}} = -\mathbf{p}_r \quad (14)$$

$$\frac{\partial H}{\partial \mathbf{a}} = 0 \Rightarrow \mathbf{a} = -\mathbf{p}_v \quad (15)$$

For fixed terminal conditions, the costates at  $t_f$  are non-zero. Integrating the costate equations yields

$$\mathbf{p}_r = \mathbf{p}_r(t_f) \quad (16)$$

$$\mathbf{p}_v = \mathbf{p}_v(t_f) + (t_f - t)\mathbf{p}_r(t_f) \quad (17)$$

and the control acceleration can be found as

$$\mathbf{a} = -\mathbf{p}_v = -\mathbf{p}_v(t_f) - (t_f - t)\mathbf{p}_r(t_f) \quad (18)$$

The state equations can be integrated as

$$\mathbf{v} = \mathbf{v}_f + (t_f - t)\mathbf{p}_v(t_f) + \frac{1}{2}(t_f - t)^2 \mathbf{p}_r(t_f) - (t_f - t)\mathbf{g} \quad (19)$$

$$\mathbf{r} = \mathbf{r}_f - (t_f - t)\mathbf{v}_f - \frac{1}{2}(t_f - t)^2 \mathbf{p}_v(t_f) - \frac{1}{6}(t_f - t)^3 \mathbf{p}_r(t_f) + \frac{1}{2}(t_f - t)^2 \mathbf{g} \quad (20)$$

Defining the time-to-go as  $t_{go} = t_f - t$ , the costates can be found as

$$\mathbf{p}_r(t_f) = \frac{12}{t_{go}^3} (\mathbf{r} - \mathbf{r}_f) + \frac{6}{t_{go}^2} (\mathbf{v} + \mathbf{v}_f) \quad (21)$$

$$\mathbf{p}_v(t_f) = - \frac{6}{t_{go}^2} (\mathbf{r} - \mathbf{r}_f) - \frac{2}{t_{go}} (\mathbf{v} + 2\mathbf{v}_f) + \mathbf{g} \quad (22)$$

Finally, the optimal control acceleration is found as<sup>1,2</sup>

$$\mathbf{a} = \frac{6[\mathbf{r}_f - (\mathbf{r} + t_{go}\mathbf{v}_f)]}{t_{go}^2} + \frac{4(\mathbf{v}_f - \mathbf{v})}{t_{go}} - \mathbf{g} \quad (23)$$

Note that  $\mathbf{g}$  is assumed to be constant here.

### Zero-Effort-Miss/Zero-Effort-Velocity (ZEM/ZEV) Guidance

The CTVG algorithm above was derived for a constant gravitational acceleration. A similar guidance law can be found for the case when the gravitational acceleration is an explicit function of time. Although in practice it is actually a function of the time-varying position, it is still useful to derive this guidance law since the gravitational acceleration does not change much over a one-day terminal-phase mission.

By following the same derivation procedure above, but with a time-varying gravitational acceleration, we can find

$$\mathbf{a} = \frac{6[\mathbf{r}_f - (\mathbf{r} + t_{go}\mathbf{v})]}{t_{go}^2} - \frac{2(\mathbf{v}_f - \mathbf{v})}{t_{go}} + \frac{6 \int_t^{t_f} (\tau - t)\mathbf{g}(\tau) d\tau}{t_{go}^2} - \frac{4 \int_t^{t_f} \mathbf{g}(\tau) d\tau}{t_{go}} \quad (24)$$

Let the zero-effort-miss (ZEM) be the position offset at the end of the mission if no more acceleration is applied. Also let the zero-effort-velocity (ZEV) be the end-of-mission velocity offset with no applied acceleration. Then, we have the ZEV and ZEM vectors mathematically expressed as<sup>4,5</sup>

$$\mathbf{ZEV} = \mathbf{v}_f - \left[ \mathbf{v} + \int_t^{t_f} \mathbf{g}(\tau) d\tau \right] \quad (25)$$

$$\mathbf{ZEM} = \mathbf{r}_f - \left[ \mathbf{r} + \mathbf{v}t_{go} + \int_t^{t_f} (t_f - \tau)\mathbf{g}(\tau) d\tau \right] \quad (26)$$

Finally, the optimal control acceleration of the CTVG or the ZEM/ZEV guidance is expressed as<sup>4,5</sup>

$$\mathbf{a} = \frac{6}{t_{go}^2}\mathbf{ZEM} - \frac{2}{t_{go}}\mathbf{ZEV} \quad (27)$$

### Computation of Time-to-Go for CTVG

In some missions, the final time  $t_f$  is a specified parameter, and the guidance law must use that time when computing time-to-go. However, for missions when the final time is not important, a final time that minimizes the performance index can be found. Reference 19 shows how to compute the time-to-go to minimize the performance index. The mission time-to-go is given as

$$t_{go} = \begin{cases} \tau & B^2 - 4AC > 0 \text{ and } B > 0 \\ \text{no solution} & \text{otherwise} \end{cases} \quad (28)$$

where

$$\begin{aligned} \tau &= \frac{-B - \sqrt{B^2 - 4AC}}{2A} \\ A &= \mathbf{v}^T \mathbf{v} + \mathbf{v}_f^T \mathbf{v} + \mathbf{v}_f^T \mathbf{v}_f \geq 0 \\ B &= -6(\mathbf{r}_f - \mathbf{r})^T (\mathbf{v} + \mathbf{v}_f) \\ C &= 9(\mathbf{r}_f - \mathbf{r})^T (\mathbf{r}_f - \mathbf{r}) \geq 0 \end{aligned}$$

This gives a local minimum solution for the performance index. There also exists a local maximum. For the “no-solution” case, the performance index is monotonically decreasing, meaning that longer mission times lead to a smaller performance index.

### Free-Terminal-Velocity Guidance (FTVG)

If the terminal velocity vector is not specified, the boundary conditions for unconstrained final velocity give  $\mathbf{p}_v(t_f) = 0$ , thus Eq. (17) becomes

$$\mathbf{p}_v = t_{go}\mathbf{p}_r(t_f) \quad (29)$$

The optimal control relation, Eq. (18), becomes

$$\mathbf{a} = -\mathbf{p}_v = -t_{go}\mathbf{p}_r(t_f) \quad (30)$$

The states are integrated as

$$\mathbf{v} = \mathbf{v}_f + \frac{t_{go}^2}{2}\mathbf{p}_r(t_f) - t_{go}\mathbf{g} \quad (31)$$

$$\mathbf{r} = \mathbf{r}_f - t_{go}\mathbf{v}_f - \frac{t_{go}^3}{6}\mathbf{p}_r(t_f) + \frac{t_{go}^2}{2}\mathbf{g} \quad (32)$$

The FTVG law is finally found as<sup>1</sup>

$$\mathbf{a} = \frac{3}{t_{go}^2}(\mathbf{r}_f - \mathbf{r}) - \frac{3}{t_{go}}\mathbf{v} - \frac{3}{2}\mathbf{g} \quad (33)$$

where  $\mathbf{g}$  is assumed be constant here.

### Computation of Time-to-Go for FTVG

Similar to the time-to-go computation for CTVG, Reference 19 gives the time to go for FTVG as

$$\tau = \frac{2 \|\mathbf{r}_f - \mathbf{r}\|}{\|\mathbf{v}\|} \left( \cos \theta - \sqrt{\cos^2 \theta - \frac{3}{4}} \right), \quad -30^\circ < \theta < 30^\circ \quad (34)$$

where  $\theta$  is the angle between  $\mathbf{v}$  and  $\mathbf{r}_f - \mathbf{r}$ .

Again there is a constraint for a finite solution existing, and outside that constraint increasingly large final times lead to an increasingly smaller performance index value.

### Intercept-Angle-Control Guidance (IACG)

Both CTVG and FTVG command the final position. The terminal velocity vector can be commanded, as in CTVG, or free as in FTVG. Consider an orthogonal coordinate system with the first component ( $\mathbf{e}_1$ ) in the direction of the desired terminal velocity, and the other two components ( $\mathbf{e}_2$  and  $\mathbf{e}_3$ ) perpendicular to this direction. Commanding the terminal velocity direction is equivalent to a combination of leaving the final velocity in the  $\mathbf{e}_1$ -direction free, and commanding the final velocity in the  $\mathbf{e}_2$ - and  $\mathbf{e}_3$ -directions to be zero. Using the FTVG law in the  $\mathbf{e}_1$ -direction, and the CTVG law in the  $\mathbf{e}_2$ - and  $\mathbf{e}_3$ -directions gives

$$\mathbf{a} = \left( \frac{3(r_{f1} - r_1)}{t_{go}^2} - \frac{3v_1}{t_{go}} - \frac{3g_1}{2} \right) \mathbf{e}_1 + \left( \frac{6(r_{f2} - r_2)}{t_{go}^2} - \frac{4v_2}{t_{go}} - g_2 \right) \mathbf{e}_2 + \left( \frac{6(r_{f3} - r_3)}{t_{go}^2} - \frac{4v_3}{t_{go}} - g_3 \right) \mathbf{e}_3 \quad (35)$$

where

$$\begin{aligned} r_i &= \mathbf{r}^T \mathbf{e}_i ; & r_{fi} &= \mathbf{r}_f^T \mathbf{e}_i \\ v_i &= \mathbf{v}^T \mathbf{e}_i ; & g_i &= \mathbf{g}^T \mathbf{e}_i \end{aligned}$$

This guidance law can also be expressed in terms of vectors  $\mathbf{r}$ ,  $\mathbf{r}_f$ ,  $\mathbf{v}$ , and  $\mathbf{g}$  as<sup>19</sup>

$$\mathbf{a} = (3\mathbf{e}_1\mathbf{e}_1^T + 6\mathbf{e}_2\mathbf{e}_2^T + 6\mathbf{e}_3\mathbf{e}_3^T) \frac{\mathbf{r}_f - \mathbf{r}}{t_{go}^2} - (3\mathbf{e}_1\mathbf{e}_1^T + 4\mathbf{e}_2\mathbf{e}_2^T + 4\mathbf{e}_3\mathbf{e}_3^T) \frac{\mathbf{v}}{t_{go}} - \left( \frac{3}{2}\mathbf{e}_1\mathbf{e}_1^T + \mathbf{e}_2\mathbf{e}_2^T + \mathbf{e}_3\mathbf{e}_3^T \right) \mathbf{g} \quad (36)$$

For the case of small light-of-sight angles, the CTVG law gives the acceleration command perpendicular to the line-of-sight direction as

$$a = 4V_c \dot{\lambda} + \frac{2V_c(\lambda - \lambda_f)}{t_{go}} - g \quad (37)$$

where  $\lambda_f$  is the desired terminal line-of-sight angle and  $g$  is the component of the gravitational acceleration perpendicular to the line-of-sight. Equation (37) is known as the PN guidance law with impact angle control.<sup>3</sup>

## PRACTICAL GUIDANCE LAW IMPLEMENTATION

In order to use the guidance laws for the asteroid intercept problem, the guidance laws can be cast in the heliocentric target-relative coordinates shown in Figure 1. The terminal guidance laws described in the preceding sections were found assuming a constant gravitational acceleration. In general, this will not be the case, rather it is a function of position and thus changes over time as position changes. During the terminal mission phase, the spacecraft and the target are very close to each other compared to the distance to the sun, and the sun's gravitational acceleration is constant to a good approximation. It is, however, updated with the rest of the guidance law as the positions and velocities of target and spacecraft are needed for the guidance law.

For practical applications for the case with  $\vec{G} = \vec{G}(\vec{R}, t)$ , the constrained-terminal-velocity guidance (CTVG) law with gravity compensation has the following form:<sup>1,2</sup>

$$\vec{A}_c = -\frac{6(\vec{R} + t_{go}\vec{V}_f)}{t_{go}^2} + \frac{4(\vec{V}_f - \vec{V})}{t_{go}} - \vec{G}(\vec{R}, t) \quad (38)$$

where  $\vec{V} = \dot{\vec{R}}$  and  $\vec{G}$  is described by Equation (5).

Similarly, we propose the free-terminal-velocity guidance (FTVG) law with gravity compensation expressed as

$$\vec{A}_c = -\frac{3}{t_{go}^2} \vec{R} - \frac{3}{t_{go}} \vec{V} - \frac{3}{2} \vec{G}(\vec{R}, t) \quad (39)$$

and the intercept-angle-control guidance (IACG) law with gravity compensation of the form

$$\vec{A}_c = - (3\mathbf{e}_1\mathbf{e}_1^T + 6\mathbf{e}_2\mathbf{e}_2^T + 6\mathbf{e}_3\mathbf{e}_3^T) \frac{\vec{R}}{t_{go}^2} - (3\mathbf{e}_1\mathbf{e}_1^T + 4\mathbf{e}_2\mathbf{e}_2^T + 4\mathbf{e}_3\mathbf{e}_3^T) \frac{\vec{V}}{t_{go}} - \left( \frac{3}{2}\mathbf{e}_1\mathbf{e}_1^T + \mathbf{e}_2\mathbf{e}_2^T + \mathbf{e}_3\mathbf{e}_3^T \right) \vec{G} \quad (40)$$

where  $\vec{G} = \vec{G}(\vec{R}, t)$ .



## RESULTS AND SIMULATIONS

The guidance laws for the three mission scenarios (specified final velocity, free final velocity, or specified intercept angle) were simulated with a 4<sup>th</sup>-order Runge Kutta numerical integration scheme. The baseline spacecraft is assumed to be a 1000-kg interceptor with 10-N thrusters. It is assumed that positions and velocities of the target and spacecraft are available with no errors.

The terminal phase begins with Apophis at perihelion, with nominal impact time 24 hours later. The approach angle and relative impact velocity are sufficient to determine the initial conditions for a spacecraft on course to intercept after 24 hours. Missions will usually be specified by these two quantities, with reference to other angles or characteristics as needed.

### Hypervelocity Intercept

*Initial Conditions* The initial conditions for high-speed impact are those used in previous studies,<sup>17,18</sup> which allows direct comparison of the performance of the optimal guidance laws considered in this paper with guidance laws from previous studies. Absent any active control, these initial conditions result in a miss distance of 40,000 km. These initial conditions are given in Table 1.

**Table 1. High-Speed Initial Conditions.**

body	x position, km	y position, km	x velocity, km/s	y velocity, km/s
Target	$111.626 \times 10^6$	0	0	37.6301
Interceptor	$111.627 \times 10^6$	$936.358 \times 10^3$	0	26.7926

*Simple Intercept* The first mission type is the simple intercept. All three guidance laws considered in this paper, FTVG, CTVG, and IACG, are used. For the high-speed initial conditions, all three algorithms achieve intercept without problem.

It is interesting to note that FTVG actually yields a somewhat higher performance index than PN guidance. This can be attributed to the FTVG derivation with gravity compensation, while in operation the gravitational acceleration is actually updated at each step. Additionally, the standard initial conditions are not necessarily ideal for a time-of-flight of exactly one day, and in fact for the standard initial conditions, a flight time only 20 seconds shorter improves this performance index. Since FTVG commands a time of flight, selecting a final time too high or too low will use more fuel, as the spacecraft burns some fuel to speed up or slow down a little bit rather than changing mostly lateral velocity.

CTVG and IACG both require the same  $\Delta V$  for both the nominal 24-hour flight time, and their respective calculated best flight times. CTVG requires a specified impact velocity, which is chosen as the initial closing velocity. IACG ends with a final impact within 1 m/s of the initial closing velocity. CTVG and IACG also use nearly the same amount of  $\Delta V$ .

Table 2 shows a comparison of the FTVG, CTVG, and IACG laws with various other guidance laws considered in previous papers.<sup>17,18</sup> Both the nominal mission time of one day and the optimal mission time for these initial conditions, one day minus 20 seconds, are shown.

Time histories of the performance index  $J$ ,  $\Delta V$ , line-of-sight angle, line-of-sight rate, acceleration commands, and closing velocities for PN, FTVG, CTVG, and IACG guidance are shown in

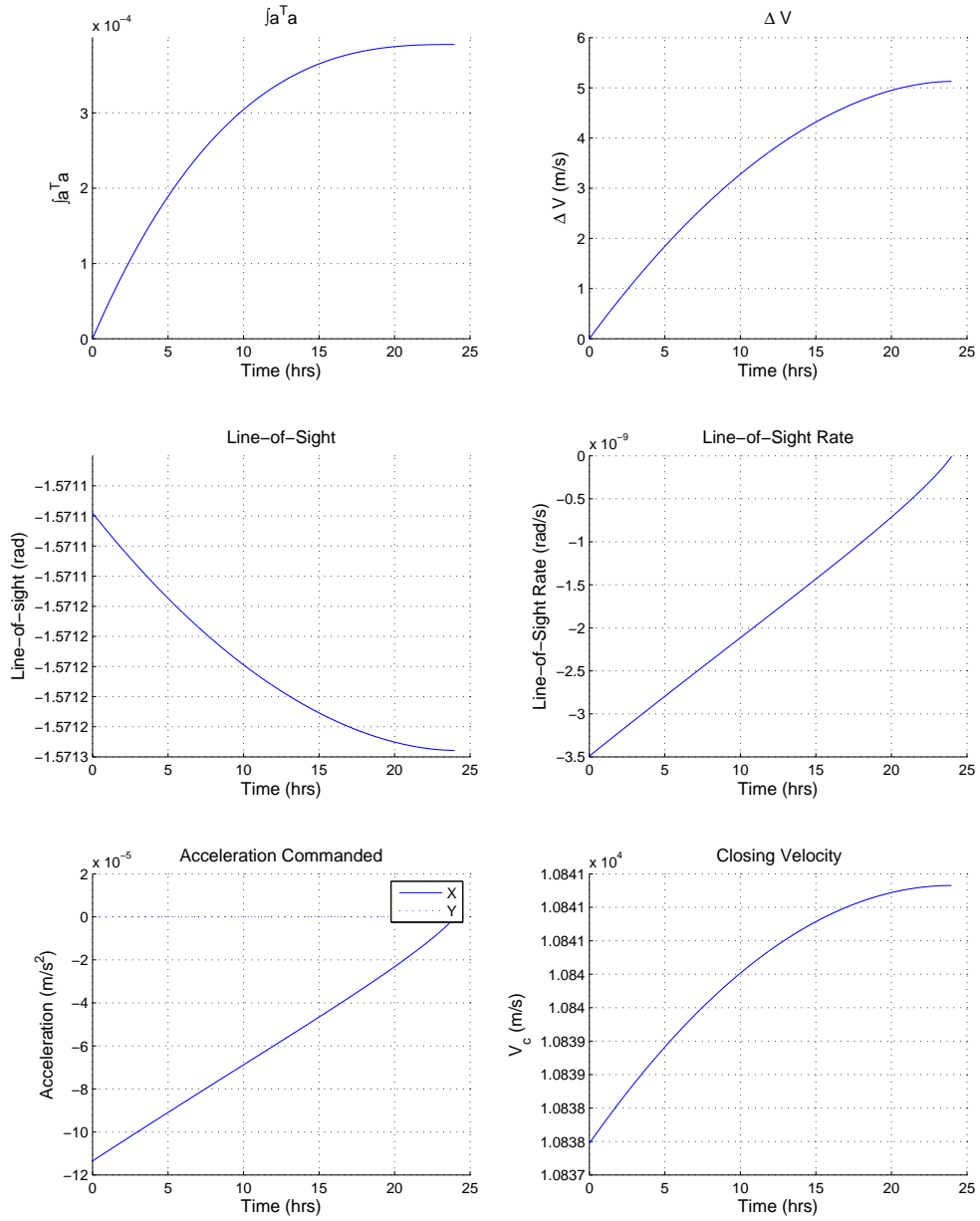
**Table 2. Performance comparison of various guidance laws**

Guidance Laws	$\Delta V$ , m/s	ZEM, m
PN	5.67	4.27
Augmented PN	5.55	3.43
Pulsed PN	4.72	4.37
Pulsed Augmented PN	4.23	3.34
Predictive Impulsive	4.34	3.39
Kinematic Impulsive	4.34	4.87
Orbit Determination	5.81	6.11
FTVG with nominal $t_f$	8.35	5.23
FTVG with optimal $t_f$	5.68	4.27
CTVG	6.42	3.84
IACG	6.48	4.54

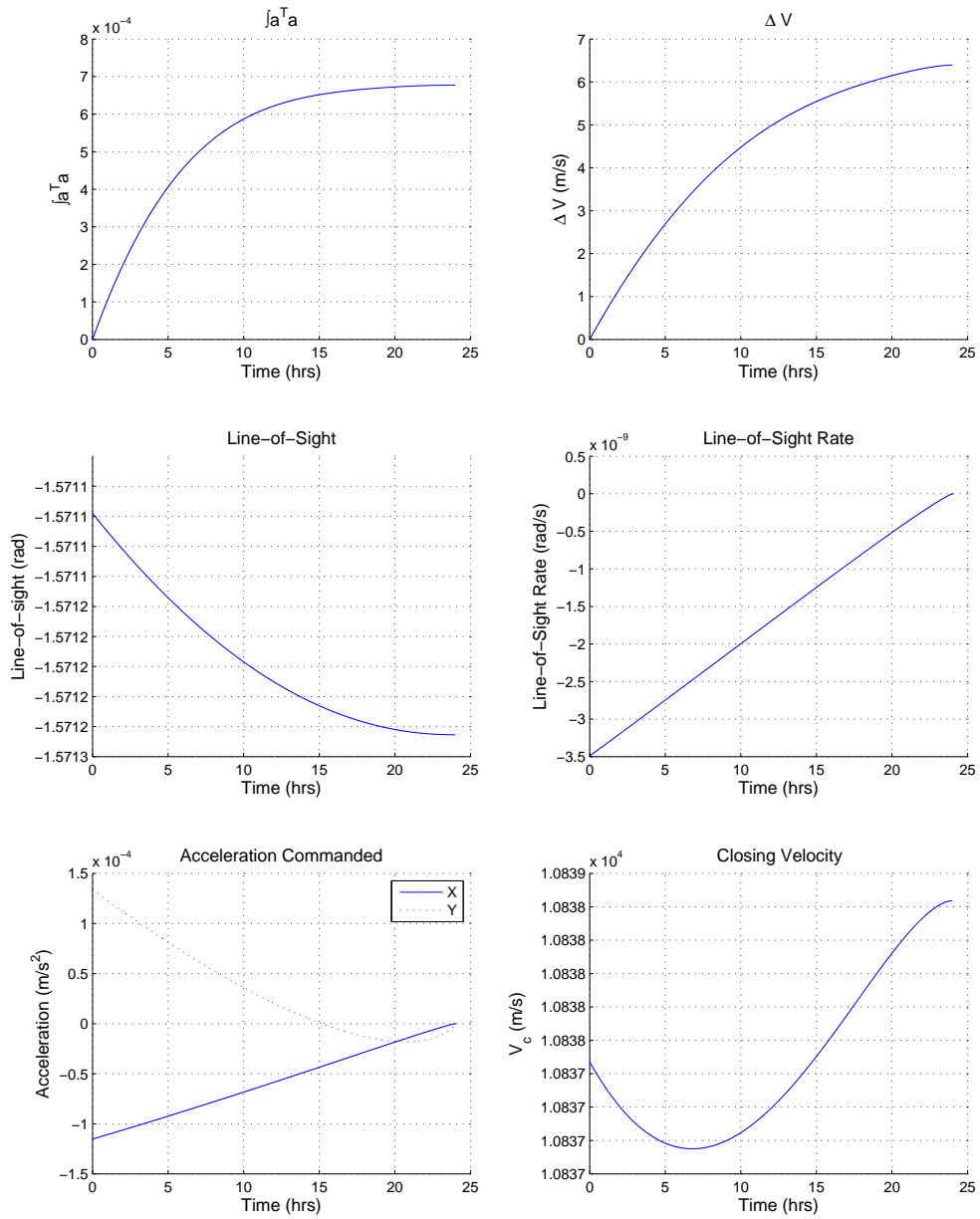
Figures 2 - 5. Although the final impact conditions are similar for all of the guidance laws, these figures show some of the differences in how each guidance law commands impact.

The FTVG guidance law minimizes the integral of the acceleration squared for a given time of flight. The selection of this final flight time is important, as it is shown above that changing the time of flight by only 20 seconds over a day can have a noticeable impact on fuel usage. To demonstrate this effect, perfect initial conditions were simulated for a range of total mission times. The perfect initial conditions achieve impact at one day of flight time, so adjusting this flight time will require more control effort. Figures 6(a) and 6(b) show the integral of the acceleration squared,  $J$ , for a range of flight times, as well as the  $\Delta V$  required. Note that the control law is derived to minimize  $J$ . The  $\Delta V$  required is of more interest for mission design, and it is seen that although the minimum  $\Delta V$  does not quite coincide with the minimum  $J$ , they are close enough that the control that minimizes  $J$  will provide low  $\Delta V$  usage. It is also worth noting that the computed “best” mission time is exactly one day. The performance index value  $J$  is nearly minimal here, though  $\Delta V$  increases from its minimum. This mismatch between the computed best time and the actual best time comes from applying results for a constant gravity field to a case where that assumption does not hold. The range of mission times shown results in unreasonable  $\Delta V$  demands for actual mission use. This range is chosen to better show the behavior of the  $J$  and  $\Delta V$  curves.

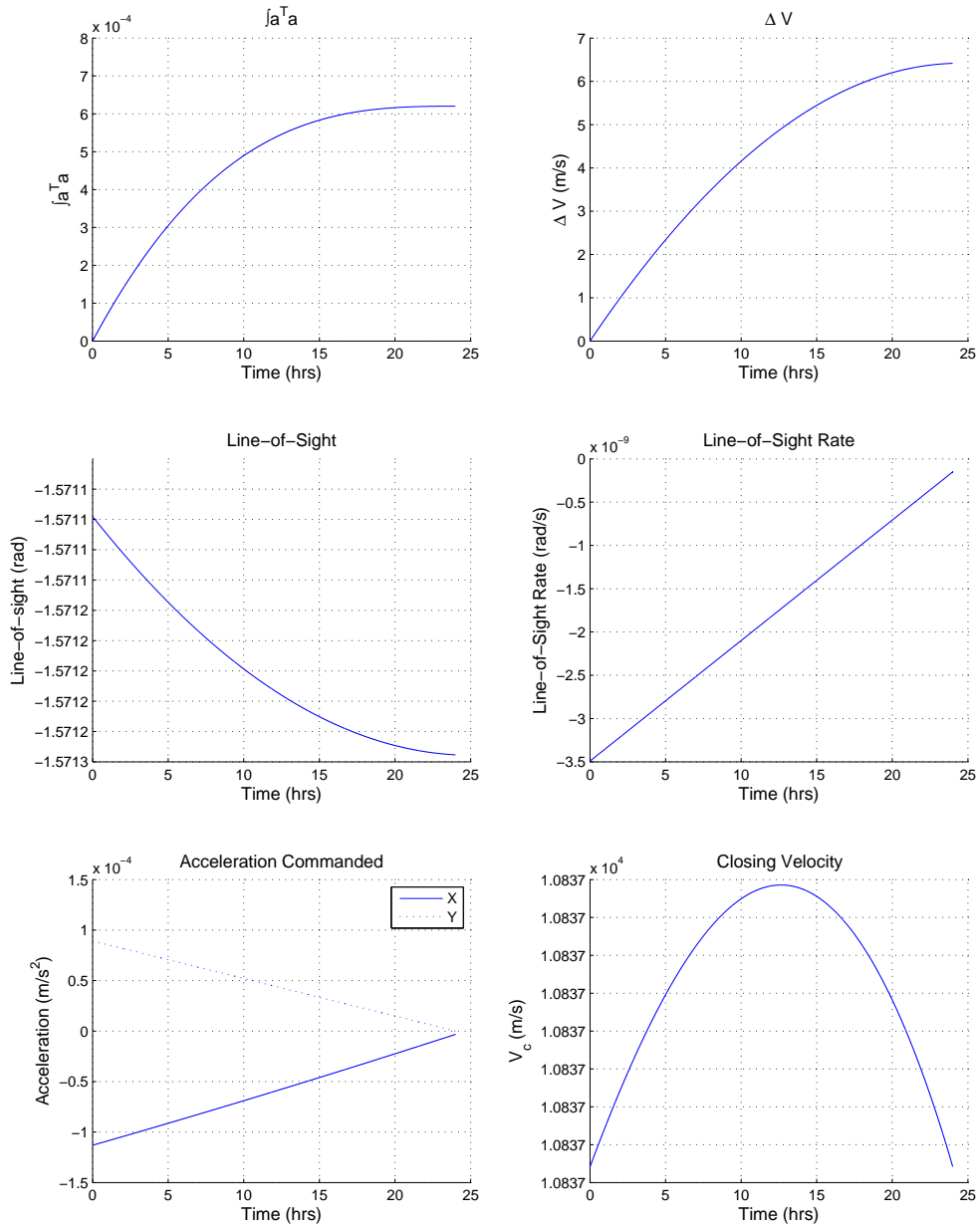
To demonstrate the sensitivity of the guidance laws to small changes in angle for a hypervelocity impact, approach angle changes from  $-2.5^\circ$  to  $+2.5^\circ$  were commanded using both CTVG and IACG for the hypervelocity impact case. The resulting  $\Delta V$  requirements are quite large, requiring more than 150 m/s for a change as small as  $0.5^\circ$ . For this mission, then, it is not feasible to significantly change the impact direction. In addition to limiting the amount of angle change possible, this example highlights the importance of having accurate knowledge of the anticipated final velocity vector. CTVG and IACG both require this direction as an input, so small errors here will lead to unnecessarily large amounts of control commanded. Figures 7(a) and 7(b) show the  $\Delta V$  required for various angle changes. Both guidance laws use the same amount of  $\Delta V$  to within less than 1 m/s, so only one curve is shown.



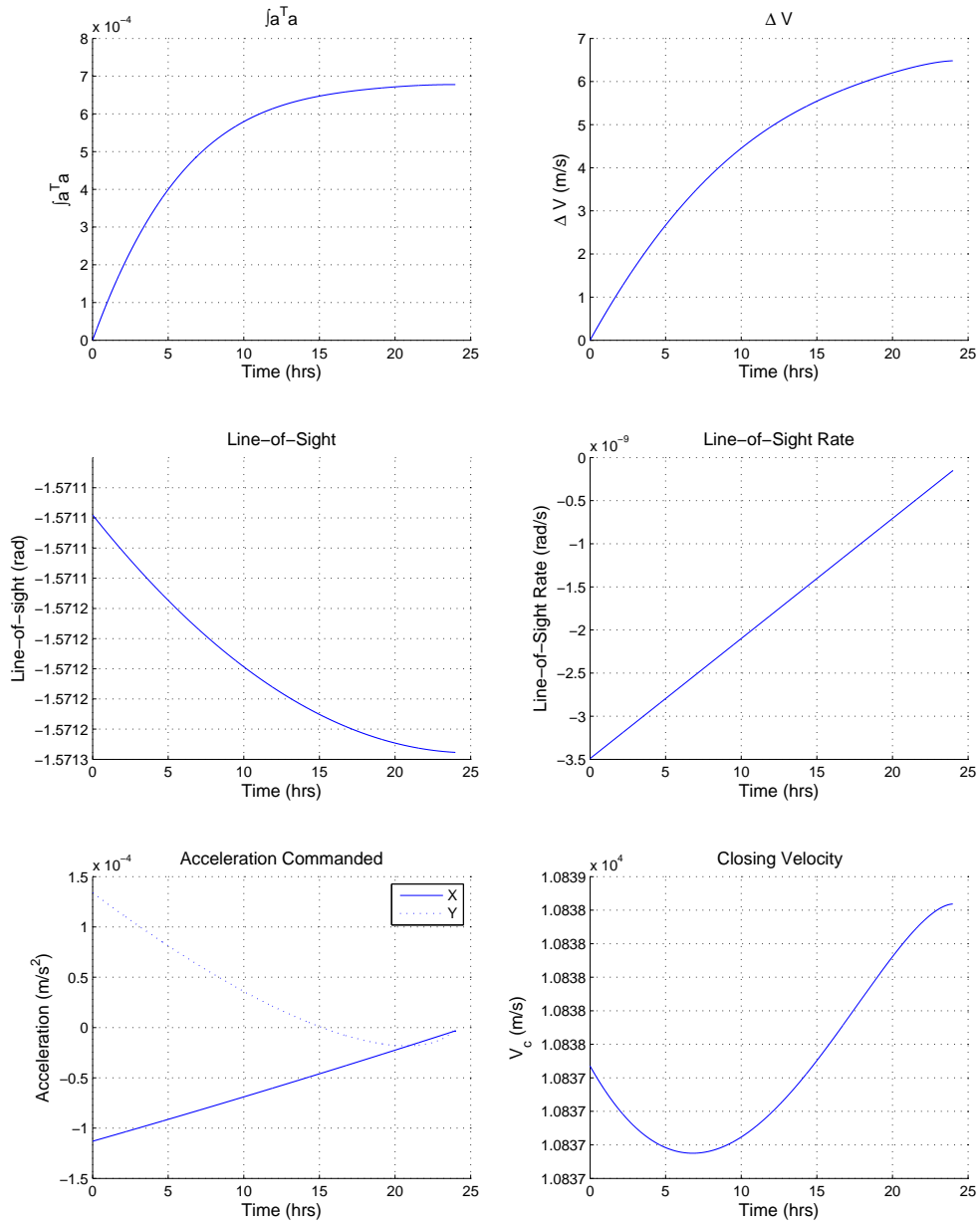
**Figure 2. Time histories of performance index,  $\Delta V$ , line-of-sight angle, line-of-sight rate, acceleration commands, and closing velocities for PN guidance.**



**Figure 3. Time histories of performance index,  $\Delta V$ , line-of-sight angle, line-of-sight rate, acceleration commands, and closing velocities for FTVG guidance.**



**Figure 4. Time histories of performance index,  $\Delta V$ , line-of-sight angle, line-of-sight rate, acceleration commands, and closing velocities for CTVG guidance.**



**Figure 5. Time histories of performance index,  $\Delta V$ , line-of-sight angle, line-of-sight rate, acceleration commands, and closing velocities for IACG guidance.**

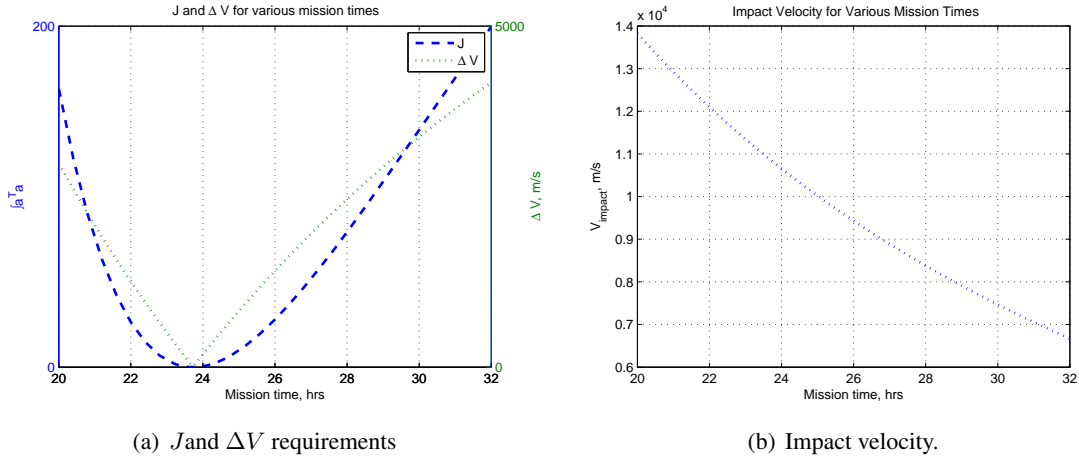


Figure 6. FTVG performance for various mission times, hypervelocity impact.

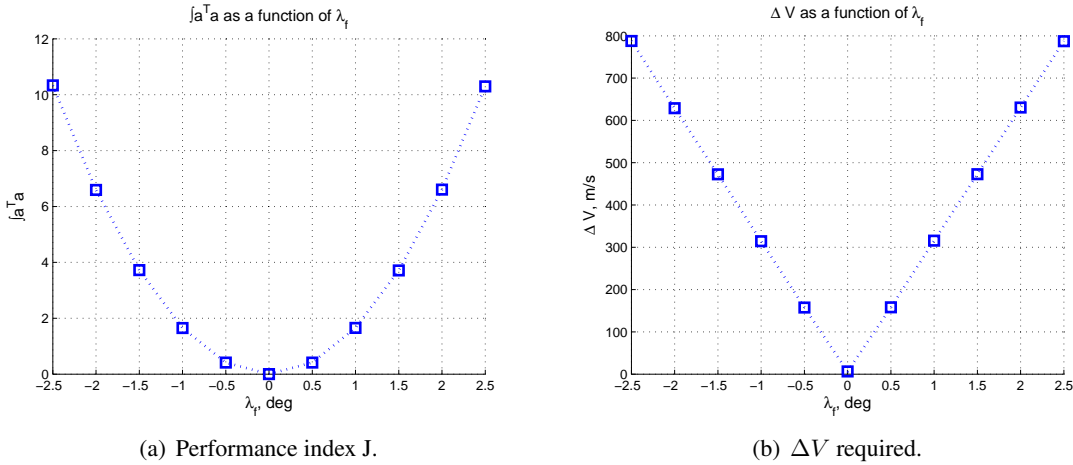


Figure 7. CTVG and IACG performance for various commanded approach angles.

### Low-speed Intercept with Intercept Angle Control

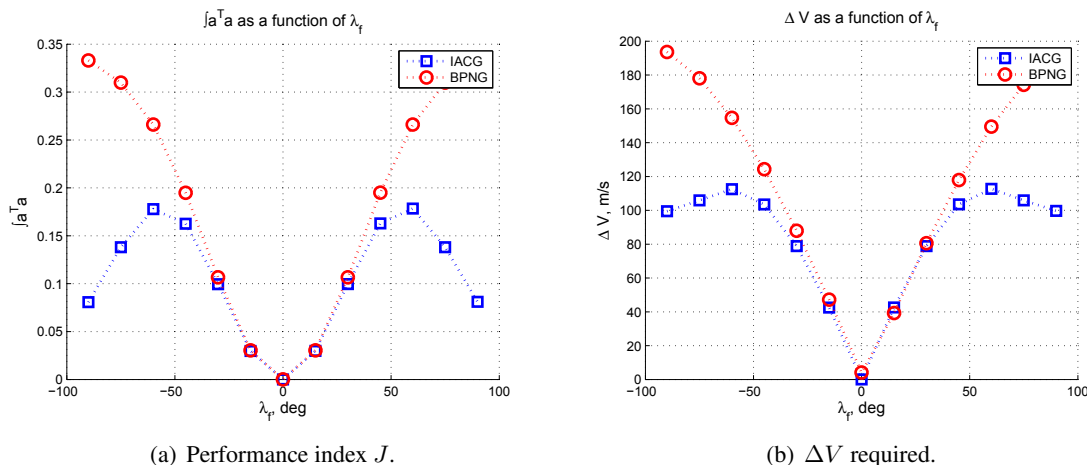
*Initial Conditions* The low-speed initial conditions are for a nominal 100 m/s impact after 24 hours of flight. These initial conditions are found by placing the spacecraft one day’s travel time ahead of the target in the target’s initial velocity direction, and displaced one kilometer outward. Without any control applied, these initial conditions result in a miss distance of 1.05 km. The low-speed initial conditions are given in Table 3.

*Controlled Impact-Angle Intercept* A previous paper<sup>18</sup> investigated PN guidance with impact angle control, referred to as a biased proportional navigation guidance (BPNG) in this paper, which is a special case of IACG. The case investigated is a nominal 100 m/s relative impact velocity with a  $0^\circ$  approach angle (a “come from behind” impact). Approach angles are commanded from  $-90^\circ$  to  $+90^\circ$ , in  $15^\circ$  increments. BPNG law does not control the mission time. For each approach angle, then, there is an associated mission time for BPNG. Using this as a baseline, the same missions (matching approach angle and total mission time) were simulated using IACG. The mission time

**Table 3. Low-Speed Initial Conditions.**

body	x position, km	y position, km	x velocity, km/s	y velocity, km/s
Target	$111.626 \times 10^6$	0	0	37.6301
Interceptor	$111.626 \times 10^6$	8640	0	37.5301

that results from the BPNG law is not necessarily the one that minimizes the performance index or the  $\Delta V$  used. Figures 8(a) and 8(b) show the performance index and the  $\Delta V$  needed for both BPNG and IACG matching the nominal BPNG mission time.



**Figure 8. IACG performance for various commanded approach angles.**

BPNG cannot control the impact velocity, and in fact the impact velocity decreases as the commanded angle change increases. IACG also does not directly control the impact velocity directly, although it can be controlled indirectly by commanding a longer or shorter flight time. Even though IACG is for the case when only the commanded angle is of importance, it is of interest to see what impact velocities result when using this law. For a given commanded angle change, there is a theoretical minimum required  $\Delta V$ , which is simply the vector difference between the starting velocity and the final velocity. This is the required  $\Delta V$  simply to change the approach direction to the required angle. Figure 9 shows the  $\Delta V$  required for various approach angles.

Both the performance index achieved and the  $\Delta V$  required increase with increasing angle changes, then decrease later. There are two reasons for this. First, the mission times are those resulting from applying BPNG, and are not necessarily the best for IACG. BPNG is based on a small-angle assumption, which is less valid as the angle change increases. This in turn leads to non-optimum mission times and performance. Second, notice that the impact velocity goes to zero at the most extreme angle changes. This means the spacecraft is slowing down significantly along the flight direction, but not changing much velocity along the transverse direction. This type of maneuver requires less  $\Delta V$  for angles closer to  $90^\circ$ .

*Controlled Impact-Velocity-Vector Intercept* The example above is continued for CTVG. For the first case, in addition to the mission time resulting from impact angle guidance, the resulting final



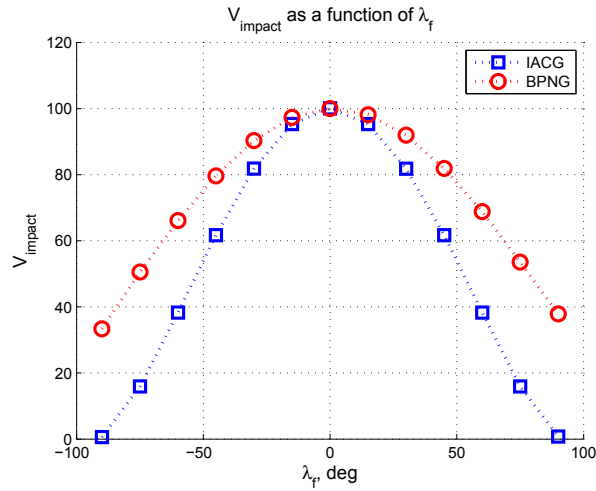


Figure 9. IACG impact velocity for various commanded approach angles.

velocity is also used as an input to the guidance system. For a given commanded terminal velocity vector, there is also a theoretical minimum required  $\Delta V$ , which is simply the magnitude of the vector difference between the starting velocity and the final velocity. This is the required  $\Delta V$  simply to change the approach direction to the required angle. Figures 10(a) and 10(b) show the performance index and the  $\Delta V$  needed for both BPNG guidance and CTVG guidance matching the nominal BPNG mission time and final velocity vector. The closing velocity is by necessity the same for BPNG and CTVG guidance, since the entire final velocity vector is matched.

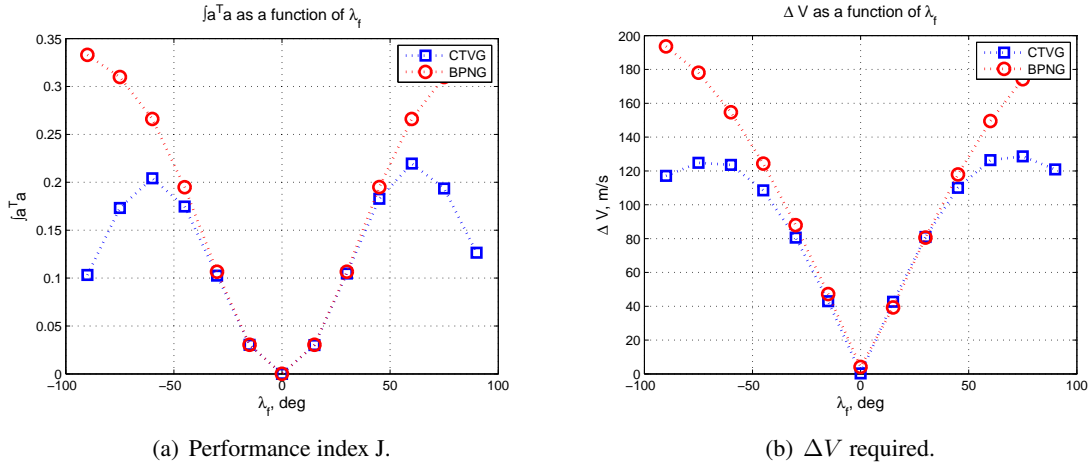


Figure 10. CTVG performance for various commanded approach angles.

Unlike impact-angle guidance, CTVG has the added ability to command any desired impact velocity. For the second case, the final closing velocity is equal to the nominal impact velocity, 100 m/s. Angles from  $-50^\circ$  to  $50^\circ$  are commanded. For the case under consideration, the equation for the best time does not have a solution for angles much past  $50^\circ$ . Additionally, the  $\Delta V$  needed for larger angles becomes quite prohibitive, and is of decreasing practical interest. Three cases are

shown. The case with a computed best final mission time is given, to show the best performance achievable. Also shown are performances when the total mission time is set to 24 hours, and to 38 hours, approximately the largest mission time when computing the minimizing time. Figure 11 shows the calculated mission time for each commanded angle.

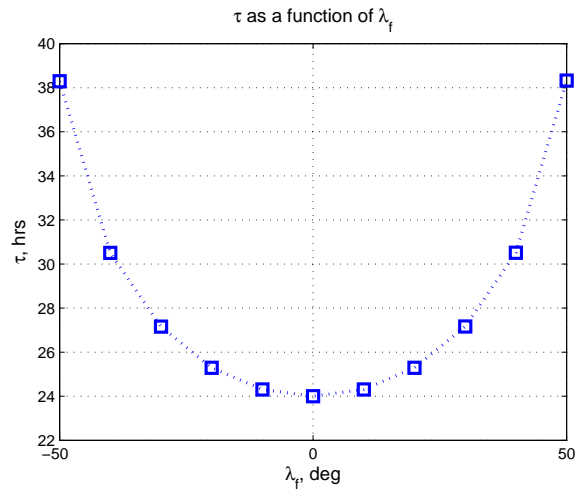


Figure 11. CTVG calculated mission time  $\tau$  for different approach angles.

Figures 12(a) and 12(b) shows the performance index and the  $\Delta V$  needed for CTVG guidance for a variety of approach angles, keeping the impact velocity as 100 m/s. The theoretical minimum  $\Delta V$  is also shown. It is seen that the performance index is in fact minimized when the best mission time is computed. However, this does not correspond to minimizing the  $\Delta V$ , which is seen to perform slightly better when keeping the mission time at exactly 24 hours. The penalty for delaying an impact is easily seen on for both performance measures.

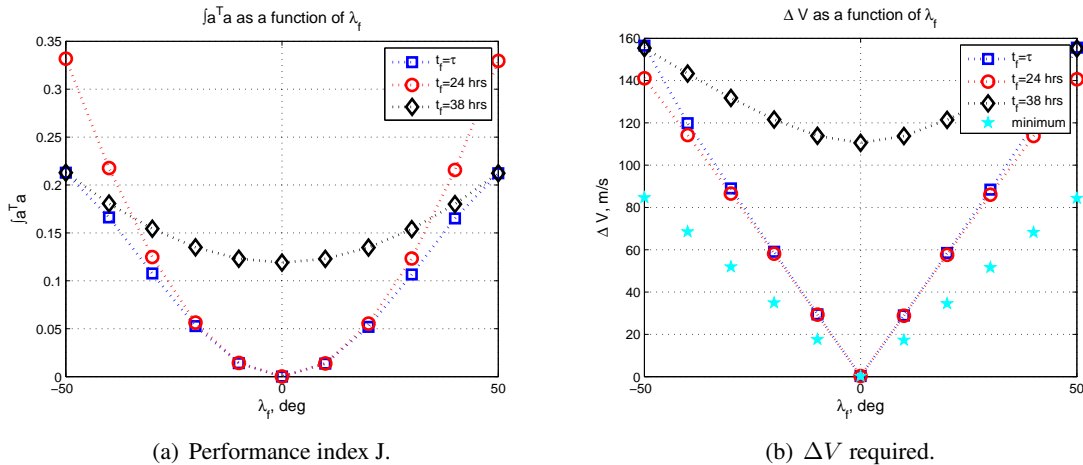


Figure 12. CTVG performance for various  $\lambda_f$  commanded approach angles.

## CONCLUSION

In this paper, three different optimal guidance algorithms, free-terminal-velocity guidance (FTVG), constrained-terminal-velocity guidance (CTVG), and intercept-angle-control guidance (IACG), were investigated. The algorithms have a variety of terminal constraints. Numerical simulations show the applicability of the guidance algorithms to the problem of intercepting a small asteroid. Issues involved with practical implementation were discussed. Further research on this topic includes investigating the precision with which the target and spacecraft orbits can be determined, and using a higher-fidelity model of the spacecraft and its engines.

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